



Integers

Suggested time: 75 min

What's important in this lesson:

You will learn how to add/subtract/multiply and divide using integers. Problems will be provided so that you may apply these skills.

Complete these steps:

1. Read the lesson portion of the package on your own.
2. Complete the exercises as they appear in the lesson.
3. Check your answers with the answer key that your teacher has.
4. Ask for help at any point during the lesson.
5. Complete the "Unit 1, Lesson 2 Integers Assignment" and submit to your teacher for evaluation.

Hand-in the following to your teacher:

1. Unit 1, Lesson 2 Integers Assignment

Questions for the teacher:

Diagnostic/Introductory Activity:
Unit 1 Lesson 2



Write a brief story to describe how the positions of the integers (-5 and +5) are related to 0. (You can write about money, temperature, elevation, etc.)

-5...0...+5

A large rectangular box with a black border. In the top-left corner, there is a colorful pencil with a yellow body, purple and blue stripes, and an orange eraser. The pencil is pointing downwards and to the right, with a blue squiggly line trailing behind it. Below the pencil, there are seven horizontal lines spaced evenly down the page, intended for writing a story.



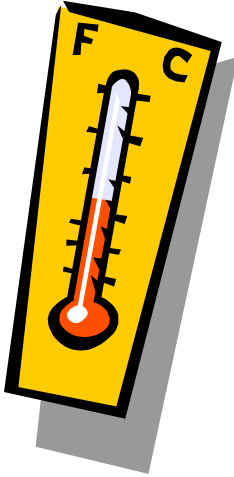
2.0 Introduction to Integers

Integers are the numbers

..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

Negative integers are less than 0.

Positive integers are greater than 0.



Integers have applications in many areas of life. We use them for temperature, elevation/depth, money, etc.



Exercise 2.0

- Write an integer for each temperature.
 - 8 degrees above 0°C
 - 35 degrees below 0°C
- Express each as an integer.
 - I owe \$5
 - a depth of 375 m
- Answer the following.

(a) $6+3$	(b) $-2+5$
(c) $-5-5+2$	(d) $7+3-6$
(e) $12-8+3-1$	(f) $-5-1-6-2$

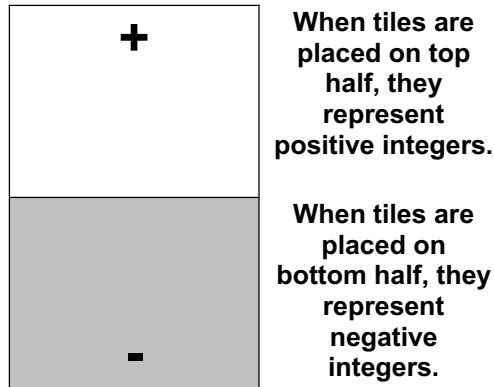


2.1 Representing Integers

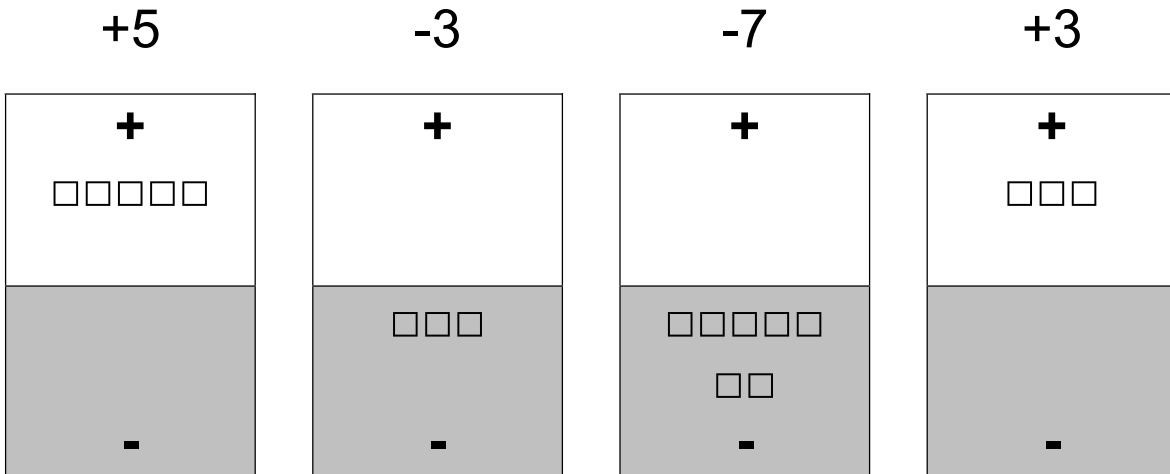
Alge-tiles can be used to represent integers. A template for alge-tiles and a mat can be found at the end of this unit.

represents 1 unit

We can use this mat to represent and perform integer operations.



We can use the tiles and the mat provided to represent integers. Here are a few examples:

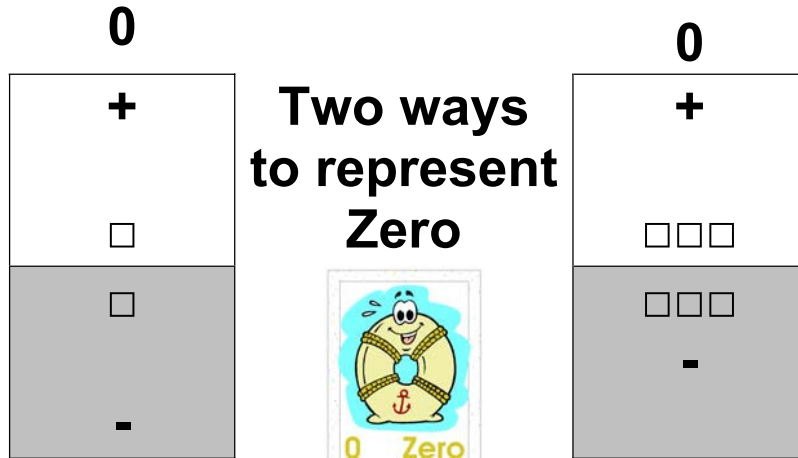




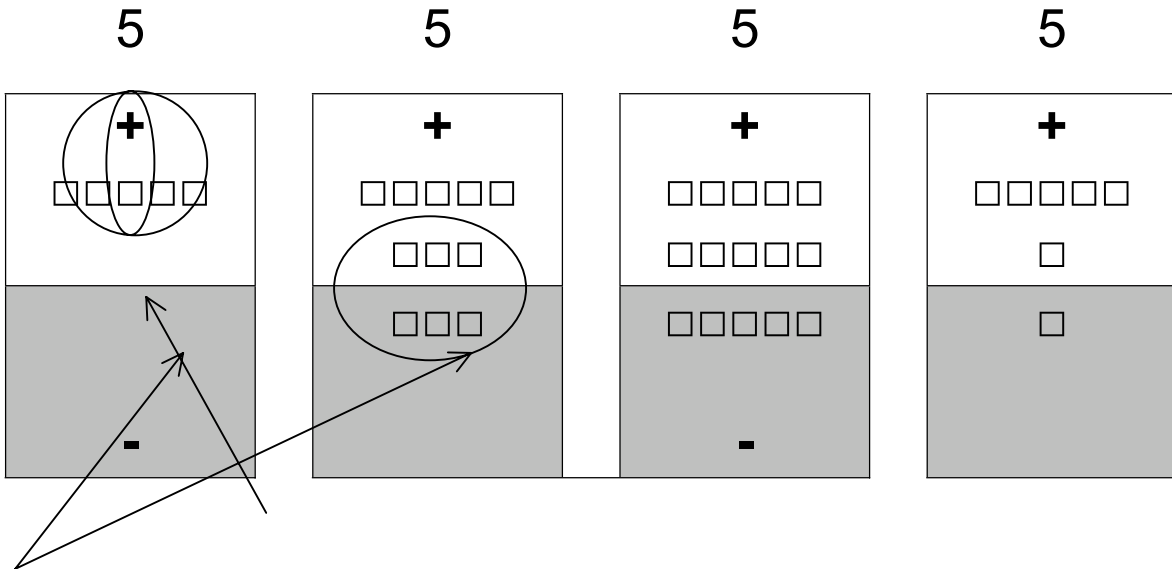
The Zero Principle

We can investigate multiple ways of representing the same number by introducing the “Zero Principle”.

We represent the “Zero Principle” by pairing a tile on the bottom of the mat with a tile on the top of the mat (+1 with -1). We could use any number of tiles to do this as long as the number of tiles on the top matches the number on the bottom.



With the Zero Principle, we can represent the same number in many different ways. Try the number 5.



**Each of these groupings
represents zero.**



Exercise 2.1

Represent the following numbers in two different ways.

(a) -4	
+	+
-	-

(b) 3	
+	+
-	-

(c) 6	
+	+
-	-

(d) -5	
+	+
-	-



2.2 Adding Integers

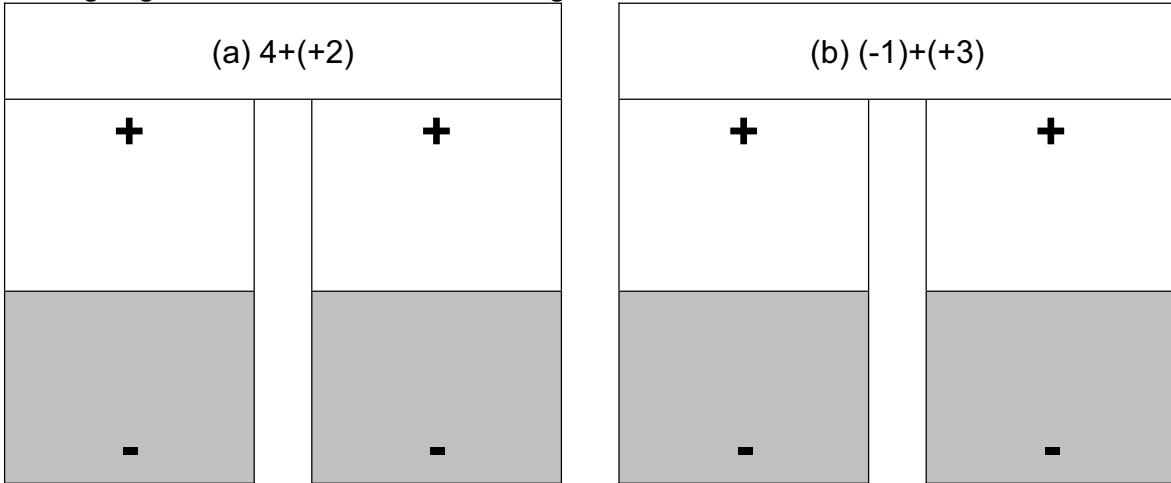
Using tiles....

<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">(a) $4 + 3$</div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px; text-align: center;"> $+$ □ □ □ □ □ □ □ </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px; background-color: #cccccc; text-align: center;">-</div> <p style="text-align: right; font-size: 24px;">= 7</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">(b) $-4 + (+3)$</div> <table border="0" style="width: 100%; text-align: center;"> <tr> <td style="border: 1px solid black; padding: 5px; width: 50%;"> $+$ □ □ □ </td> <td style="padding: 0 10px;">= 0 these can be removed</td> <td style="border: 1px solid black; padding: 5px; width: 50%;"> $+$ □ </td> </tr> <tr> <td style="border: 1px solid black; padding: 5px; background-color: #cccccc; text-align: center;"> □ □ □ □ - </td> <td></td> <td style="border: 1px solid black; padding: 5px; background-color: #cccccc; text-align: center;"> □ - </td> </tr> </table> <p style="text-align: right; font-size: 24px;">= -1</p>	$+$ □ □ □	= 0 these can be removed	$+$ □	□ □ □ □ -		□ -
$+$ □ □ □	= 0 these can be removed	$+$ □					
□ □ □ □ -		□ -					
<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">(c) $(-3) + (-1)$</div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px; text-align: center;"> $+$ </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px; background-color: #cccccc; text-align: center;"> □ □ □ □ - </div> <p style="text-align: right; font-size: 24px;">= -4</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;">(d) $(+2) + (-5)$</div> <table border="0" style="width: 100%; text-align: center;"> <tr> <td style="border: 1px solid black; padding: 5px; width: 50%;"> $+$ □ □ </td> <td style="padding: 0 10px;">= 0 these can be removed</td> <td style="border: 1px solid black; padding: 5px; width: 50%;"> $+$ </td> </tr> <tr> <td style="border: 1px solid black; padding: 5px; background-color: #cccccc; text-align: center;"> □ □ □ □ □ - </td> <td></td> <td style="border: 1px solid black; padding: 5px; background-color: #cccccc; text-align: center;"> □ □ - </td> </tr> </table> <p style="text-align: right; font-size: 24px;">= -2</p>	$+$ □ □	= 0 these can be removed	$+$	□ □ □ □ □ -		□ □ -
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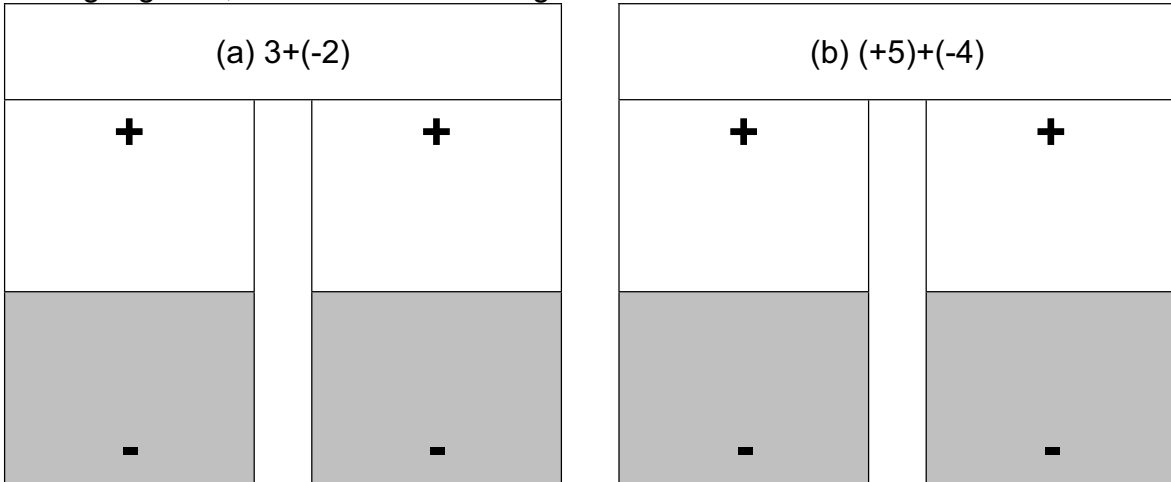
Using Common Notation...

Using Alge-tiles, examine the following.



Notice that when you **ADD a POSITIVE integer** → you **ADD**

Using Alge-tile, examine the following.



Notice that when you **ADD a NEGATIVE integer** → you **SUBTRACT**

This means we can simplify the addition of integers by using common notation.

$(a) 3+(+2)$
 $= 3+2$
 $= 5$

$(b) (-1)+(+4)$
 $= -1+4$
 $= 3$

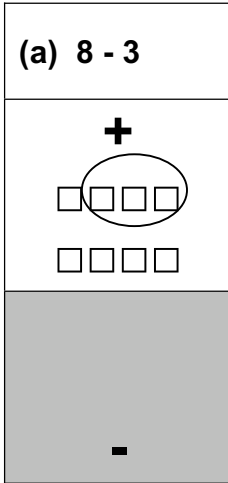
$(c) (+5)+(-2)$
 $= 5-2$
 $= 3$

$(d) (-2)+(-6)$
 $= -2-6$
 $= -8$

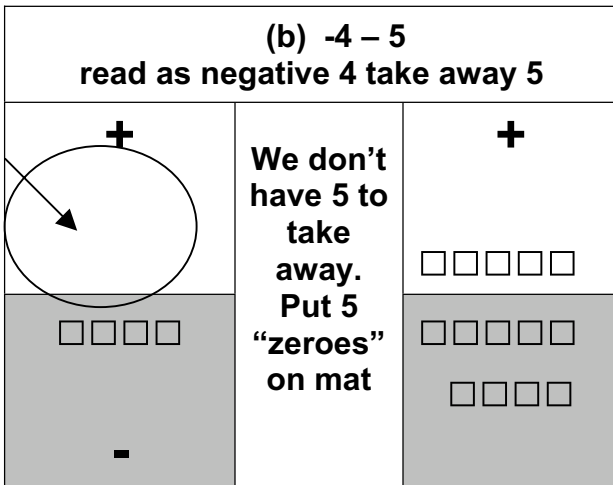


2.3 Subtracting Integers

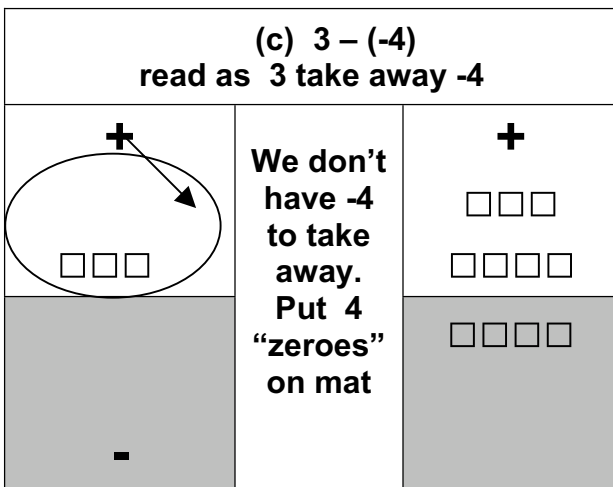
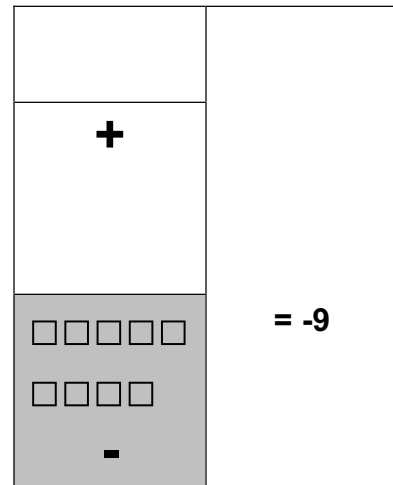
Using the tiles...



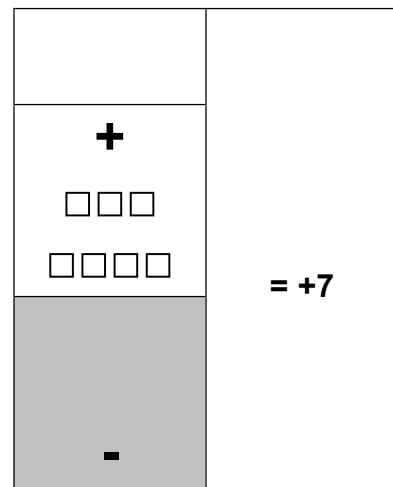
= 5



We can now take away 5 positive tiles.



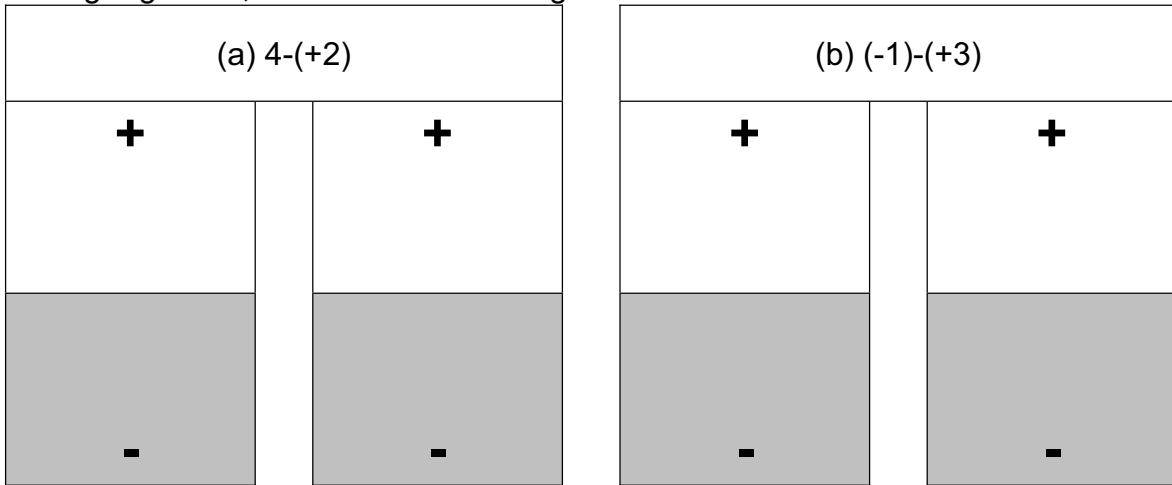
We can now take away 4 negative tiles.





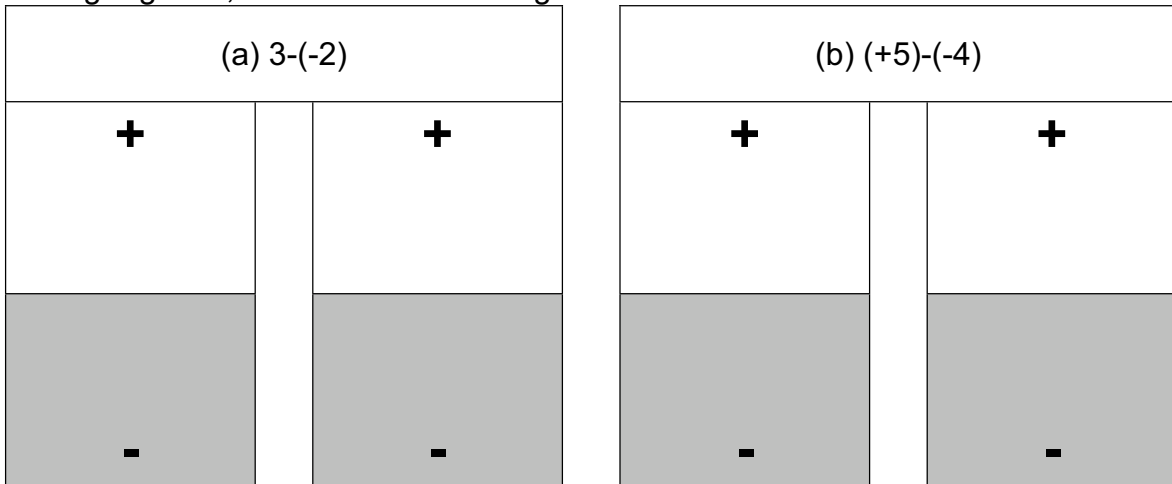
Using Common Notation...

Using Alge-tiles, examine the following.



Notice that when you ***SUBTRACT a POSITIVE integer*** → you ***SUBTRACT***

Using Alge-tile, examine the following.



Notice that when you ***SUBTRACT a NEGATIVE integer*** → you ***ADD***

We can simplify the subtraction of integers by using common notation.

$(a) \quad 3 - (+2)$
 $\quad = 3 - 2$
 $\quad = 1$

$(b) \quad (-1) - (+4)$
 $\quad = -1 - 4$
 $\quad = -5$

$(c) \quad (+5) - (-2)$
 $\quad = 5 + 2$
 $\quad = 7$

$(e) \quad (-2) - (-6)$
 $\quad = -2 + 6$
 $\quad = 4$



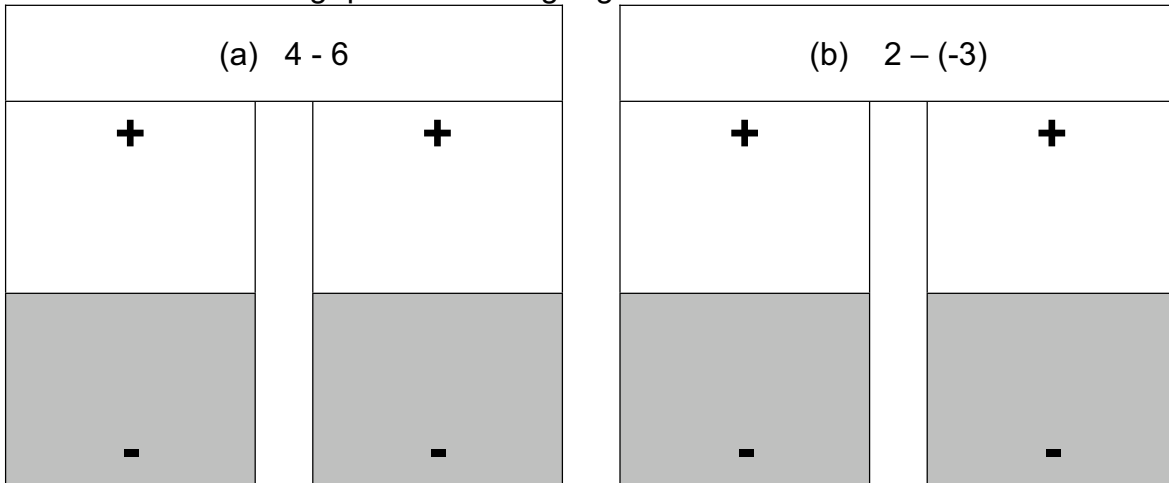
Using the money analogy...

- (a) $8 - 3$ I have \$8 and owe \$3
 $= 5$ I pay off my debt of \$3
 I have \$5 left.
- (b) $-4 - 5$ I owe Susan \$4 and I owe Jamal \$5
 $= 9$ I owe \$9 in total.
- (c) $3 - (-4)$ I have \$3 and I know that I owe Kendra \$4
 Kendra says I don't have to pay her back –
 she is taking away my \$4 debt.
 $= 7$ I have \$7 since I don't have to pay Kendra back.



Exercise 2.3

Illustrate the following questions using Alge-tiles.



Answer the following questions using common notation.

- (c) $(-2) - (+3)$
- (d) $(+9) - (-3)$
- (e) $(+6) - (+13) - (-15)$
- (f) $(+15) - (+20)$
- (g) $+27 - (+54) - (-50)$
- (h) $-7 - (-3)$
- (i) Your chequing account had a balance of \$92.75. You then wrote a cheque for \$102, which cleared. What was your new balance?



Summary of Addition & Subtraction of Integers

Add a Positive	$1+(+1)$	ADD +	$1+1=2$
Add a Negative	$1+(-1)$	SUBTRACT -	$1-1=0$
Subtract a Positive	$1-(+1)$	SUBTRACT -	$1-1=0$
Subtract a Negative	$1-(-1)$	ADD +	$1+1=2$

Determine the answer to the riddle by evaluating each question and using the answer key to complete the phrase at the bottom of the page.

How Does a Rodeo Star Get Around?

- (1) $4+(-5)$
- (2) $8 - (-2)$
- (3) $(+2)+(+5)$
- (4) $6 - (+3)$
- (5) $-10+(-3)$
- (6) $9+(-1)$
- (7) $5+(-6)+1$
- (8) $-3-(+3)$
- (9) $(-3)-(+1)$
- (10) $(-2)+(+4)$
- (11) $(-10)+(+5)+2$
- (12) $(+3)-(-6)$
- (13) $(-7)+(-2)$
- (14) $(-11)-(-2)+(-3)$

A	-6
T	-13
L	7
T	-12
S	1
C	3
E	-3
W	0
I	2
C	-9
T	-1
A	10
T	-4
H	9
A	8

7	10	1	12	8	13	2	5	9	3	11	6	4	14



2.4 Multiplying and Dividing Integers

The product/quotient of two integers with the same signs is positive.

$$(+)(+) = (+)$$

$$(-)(-) = (+)$$

The product/quotient of two integers with different signs is negative.

$$(+)(-) = (-)$$

$$(-)(+) = (-)$$

A memory trick...

$$(+)\times(+)=(+)$$

love(+) to love(+) means love(+)

$$(+)\times(-)=(-)$$

Love(+) to hate (-) means hate (-)

$$(-)\times(+)=(-)$$

Hate(-) to love(+) means hate (-)

$$(-)\times(-)=(+)$$

hate(-) to hate(-) means love(+)



Example:

$$(a) \quad (-6)(-7) = +42$$

$$(b) \quad \frac{+72}{-12} = -6$$

Exercise 2.4:

$$(a) \quad (+3)(-7)$$

$$(b) \quad -51 \div (-3)$$

$$(c) \quad (-5)(-6)$$

$$(d) \quad (+72) \div (-8)$$

$$(e) \quad (-4)(+4)$$

$$(f) \quad (+5)(+2)$$

$$(g) \quad \frac{+12}{+2}$$

$$(h) \quad \frac{-6}{+6}$$

- (i) You are the captain of a submarine. While exploring the ocean, your submarine plunges 50m per minute from the surface. What will be the depth of the submarine after 4 minutes?

**BEDMAS with Integers**

Examples: $(2+3)+(-4)(+2)$
 $= (5)+(-8)$
 $= 5-8$
 $= -3$

$$\begin{aligned} & (+6)\div(-2)+(+5) \\ & = (-3)+5 \\ & = -3+5 \\ & = 2 \end{aligned}$$

Complete.

a) $4+(+3)(+2)$

(b) $(+8)\div(-4)+(+6)$

(c) $(-12)+(4+6)$

(d) $(-1)(-3)+(+5)(-2)$

(e) $18-(+2)(-3)$

(f) $(+2)(+4)\div(10-2)$

(g) $-10+(-4)(-3)$

(h) $(+6)+(+15)\div(-5)$



Placemat for adding and subtracting algebra tiles

A large rectangular placemat with a double-line border. It is divided into two horizontal sections. The top section is white and contains a large black plus sign (+) centered in the upper half. The bottom section is shaded gray and contains a large black minus sign (-) centered in the lower half. The entire placemat is intended for students to place algebra tiles to represent mathematical operations.



Algebra tiles template for use throughout Unit 1



Integers Assignment

1. Complete.

a) $(-2) + (-7)$

b) $(-2) - (-7)$

c) $(-2) \times (-7)$

d) $(-8) - (+3)$

e) $4 - (-1)$

f) $-2 + 6$

g) $(-8) \div (-1)$

h) $\frac{6}{-2}$

i) $-8 \times (4)$

j) $-3 - 5$

k) $-3 + 5 - 2 + 1$

l) $10 - (-2)(+3)$

2.

(a) The temperature is $+5^{\circ}\text{C}$. As a storm approaches, it drops 7°C . As evening comes on, it drops another 3°C . Find the final temperature.

(b) Last week, a cold wave hit Hamilton, resulting in a temperature of 2°C below zero. At the same time, it was 27°C in Miami. How much warmer was Miami than Hamilton?



