Polynomials

Suggested time: 150 min

What's important in this lesson:
You will use algebra tiles to learn how to add/subtract polynomials. Problems are provided for you to apply these skills.

Complete these steps:

1. Read the lesson portion of the package on your own. Make use of the algebra tiles as necessary.

2. Complete the exercises as they appear in the lesson.

3. Check your answers with the answer key that your teacher has.

4. Ask for help at any point during the lesson.

5. Complete the "Unit 1, Lesson 3 Polynomials Assignment" and submit to your teacher for evaluation.

Hand-in the following to your teacher:

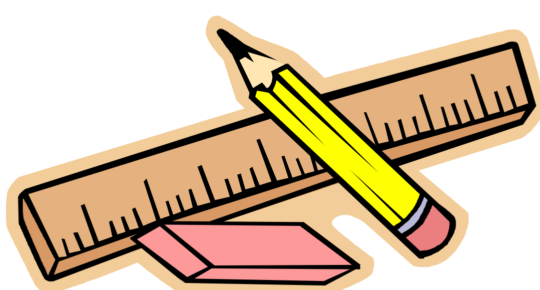
1. Unit 1, Lesson 3 Polynomials Assignment
2.
3.

Questions for the teacher:

Vocabulary Matching

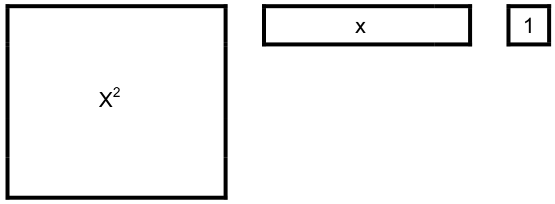
In the following table, match the word to its definition and then to an example using a straight line to connect.

WORD	DEFINITION	EXAMPLE
Constant	A letter used to represent a value that can change or vary.	**In 2x + 5, we call "5" the ___________**
Variable	A number or a variable, or the product of numbers and variables.	**In the expression 2t + 3, "t" is called a ________**
Exponent	A term that contains no variables	**The expression 5x + 3 has two _______**
Term	Use of a raised number to denote repeated multiplication of a base.	**In 3^4,the _______ is 4.**



3.0 Introduction to Polynomials

Now, let us look at the tiles you have – called algetiles. We will label them as:



We can still use the placemat from lesson 1.

Like terms in algebra have the same variables raised to the same exponents.

Examples: 2x, 5x or -7y, 12y or $6x^2$, $-x^2$

Unlike terms in algebra have different variables, or the same variable but different exponents.

Examples: 12x, 3y, $-15x^2$

A **monomial** is a number or variable or the product of numbers and variables.

Examples: 6 or x or 3x

A **polynomial** is a monomial or the sum of monomials.

A polynomial with 2 terms is called a **binomial**. Example: 4x+1

A polynomial with 3 terms is called a **trinomial.** Example: x^2-5x+7

Sorting Like Terms

Cut out the following terms.

Sort the following into groups of like terms
Paste into 5 groups

a	$2x^2$	$3a$	$3x^2$	$-9y$
$-y^2$	$4x^3$	$7y$	$7a$	$-3x^3$
$8y$	$-6a$	$12y^2$	$-a$	x^3
$5x^3$	$2y$	$6x^2$	$-4y^2$	y

Sorting Like Terms

Exercise 3.0

Word list

monomial	*Term*	*trinomial*
Coefficient	*Binomial*	*unlike terms*
Like terms	*exponent*	*variable*

Classify each of the following as a monomial, binomial or trinomial.

(a) $x – 2y$ (b) $x^2 + 3y^2 – z^2$ (c) $-32a$

In the term $-3x^2$, the -3 is the _____________ and x is a ________________.

The expression $4a^2$ has one _____________. Therefore, this expression is a _____________.

The expression $3x – 7$ has two terms. Therefore, it is a ____________. In this expression, the second term is a ___________.

The expression $4x^2 + 2x +3$ is an example of a _______________.

$2x^2$, $-3x^2$, and $11x^2$ all have the same variable and the same _______________.

Therefore, these three terms are ________ ________ . $4a^2$, -3a and 7b are ________ ________.

Simplify by collecting like terms.

(a) $5a – 7b + 8a – 11b$

$= 5a + 8a – 7b – 11b$
$= 13a –18b$

(b) $3x + 2 – 2x + 7$

(c) $7 – 2y - 10 - 3y$

(d) $3x + 7y – 5x – 10y + 20x$

(e) $7x^2 + 3x – 8x + 4x^2$

(f) $3z^3 – 2z + z^2 – 5z^3 –2z$

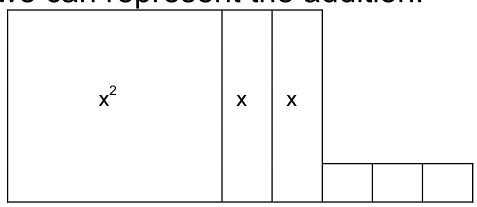
(g) $3x – 3x^2 + 3x^2 – 8x$

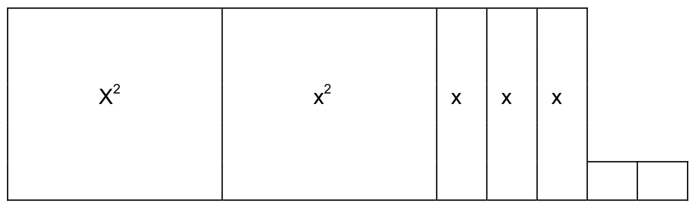
(h) $2x^2-5x+1-x^2+3x+6$

3.1 Add/Subtract Polynomials

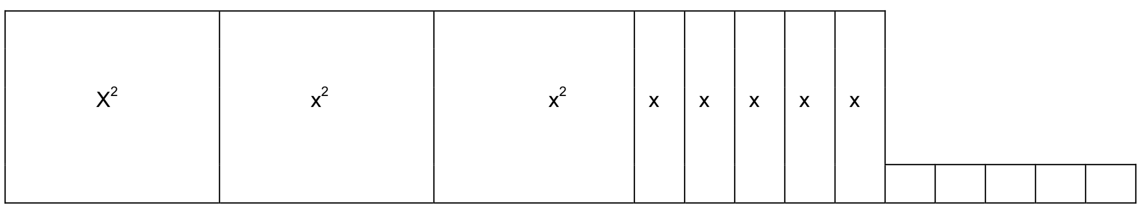
(a) $(x^2 + 2x + 3) + (2x^2 + 3x + 2)$

Using tiles, we can represent the addition:





We can collect like terms (represented by same shape of tile) and add.

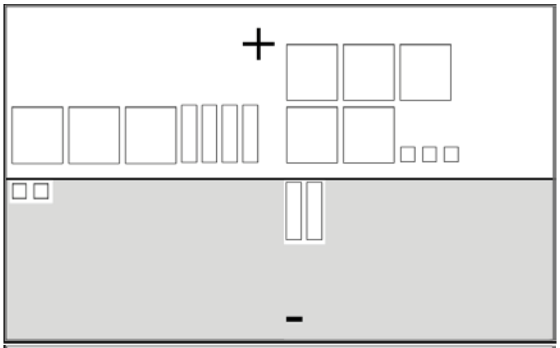


Therefore $(x^2 + 2x + 3) + (2x^2 + 3x + 2) = 3x^2 + 5x + 5$

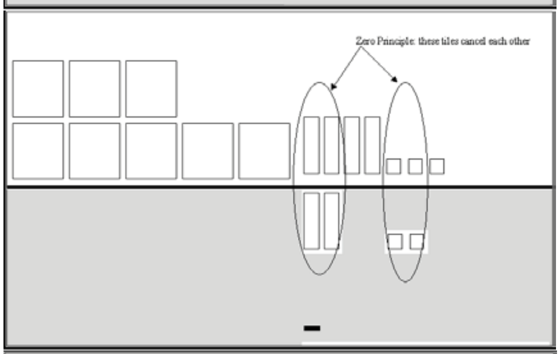
This seems simple enough when all the terms are positive; let's try one that has negative coefficients.

(b) The Question:
$(3x^2 + 4x - 2) + (5x^2 - 2x + 3)$

We can use the mat to help us keep track of positive and negative terms.



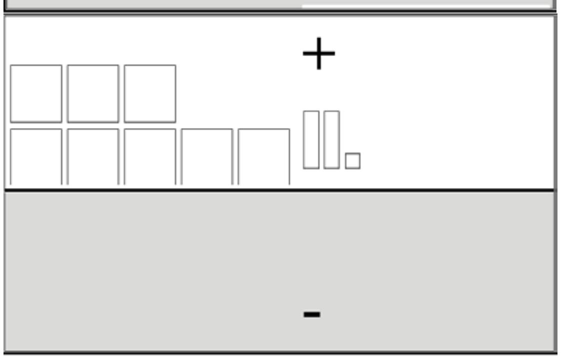
Place your tiles on the mat for each expression.



Collect like terms – group tiles that are the same shape together.

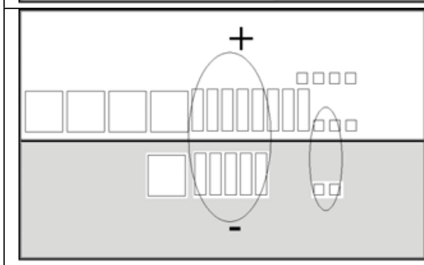
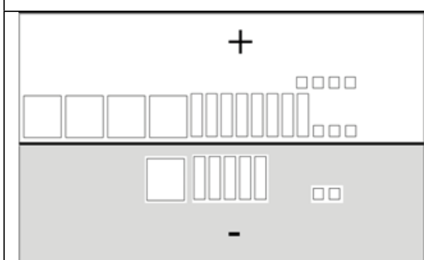
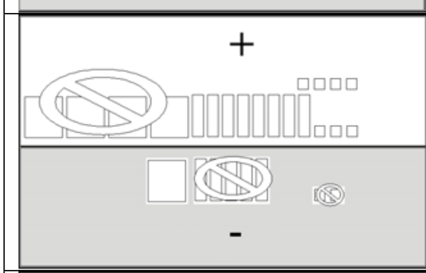
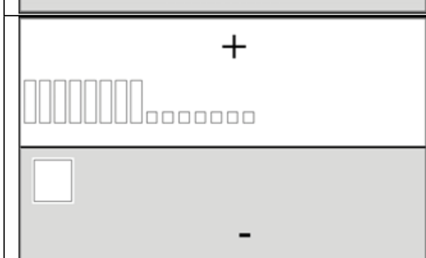
$3x^2 + 5x^2 + 4x - 2x - 2 + 3$

Tiles that appear on both sides of the mat cancel each other out because of the Zero Principle.



Regroup all remaining tiles and state the solution.

Solution:
$(3x^2 + 4x - 2) + (5x^2 - 2x + 3)$
$= 3x^2 + 4x - 2 + 5x^2 - 2x + 3$
$= 3x^2 + 5x^2 + 4x - 2x - 2 + 3$
$= 8x^2 + 2x + 1$

(c) $(3x^2 + 3x + 5) – (4x^2 – 5x – 2)$	
	Place your tiles for the first expression on the mat. We want to “take away” $4x^2 – 5x – 2$ We only have $3x^2$ We have not –x tiles and no –2 unit tiles.
	To “take away”, we must add enough “zeroes” to be able to remove the required tiles/terms. We need to take away $4x^2$, so we added a zero in x^2 tiles. We need to take away –5x, so we added 5 zeroes in x tiles. We need to take away –2, so we added 2 zeroes.
	Once we know we have added all the necessary “zeroes”, we can perform our operation. Take away $4x^2 – 5x - 2$
	Take away each term $4x^2$ -5x and –2 Regroup.
	State your solution. $-x^2 + 8x + 7$
$(3x^2 + 3x + 5) – (4x^2 – 5x – 2)$ $(3x^2 + 3x + 5) – (4x^2 – 5x – 2)$ $= 3x^2 + 3x + 5 – 4x^2 + 5x + 2$ $= 3x^2 – 4x^2 + 3x + 5x + 5 + 2$ $= -x^2 + 8x + 7$	This negative sign changes each of the signs in the following expression. Collect like terms simplify

Student Handout: Unit 1 Lesson 3

Exercise 3.1

Adding polynomials.
(4x-1)+(2x+5)
= 4x-1+2x+5
= 4x+2x-1+5
= 6x+4

Subtracting polynomials
(4x-1) – (2x+5)
= 4x-1 – 2x -5 ← Notice! You must subtract each term in the polynomial
= 4x-2x-1-5
= 2x-6

(a) $(x+1)+(2x-1)$

(b) $(2x+5)+(3x+7)$

(c) $(6y+8)+(7y-12)$

(d) $(4x^2-3x+1)+(-5x^2-6x-7)$

(e) $(3x^2-3)+(-3x^2+7x-4)$

(f) $(6x+8)-(4x-1)$

(g) $(5x^2-3x+1)-(2x^2+1)$

(h) $(6x+5y)-(-2x-2y)$

(i) $(10x+3x^2-5)-(3x^2-7+x)$

(j) $(4x+3)+(-2x-6)$

(k) $(4x+2y-6)-(2x-y+5)$

(l) $(2x-1)+(4x+7)+(x-2)$

(m) $(-4y+1)+(3y+6)-(2y-5)$

(n) $(3x-2y+4)-(2x+6y-1)-(3x+5)$

3.2 Multiplying a Monomial and a Polynomial

$$3(x + 2)$$

Monomial
(one term)
"3"

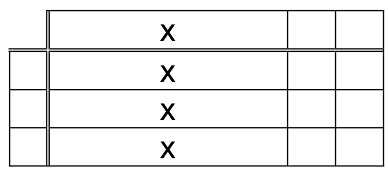
"is the same as
3 multiplied by (x + 2)"

Polynomial
(more than one term)
"(x+2)"

Expand 3(x + 2) using tiles. We can perform the multiplication visually by creating a rectangle with length of (x+2) and width of 3.

(x + 2)

3



Therefore, 3(x+2) = 3x + 6

We can perform the same multiplication using the distributive property. The number outside of the brackets must be multiplied by each term inside the brackets.

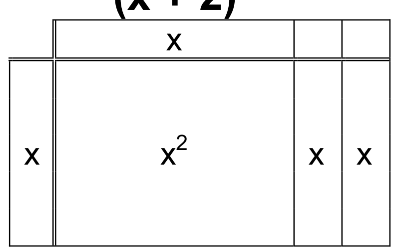
$$= 3(x) + 3(2)$$
$$= 3x + 6$$

$$x(x+2)$$

Expand x(x+2) using tiles. Notice what happens when we multiply 1 by x and x by x.

(x + 2)

x



Therefore, $x(x+2) = x^2+2x$

Meaning…

$$\mathbf{1(x) = x}$$
$$\mathbf{x(x) = x^2}$$

and similarly $\mathbf{x(x^2) = x^3}$

Exercise 3.2:

Examples:

(i) $2(x+5)$
$= 2x+10$

(ii) $-3x(2x+1)$
$= -6x^2-3x$

(a) $-4(x - 1)$

(b) $7(2x + 1)$

(c) $6(2x - 1)$

(d) $5(y + 7)$

(e) $-3(5x + 2)$

(f) $2(x^2 - 3x + 1)$

(g) $x(5x-7)$

(h) $x(x-6)$

(i) $2x(4x-1)$

(j) $-3x(2x-4)$

(k) $-5x(3x^2-2x+1)$

(l) $x(-x^2+7x-2)$

(m) $7y(2y+5)$

(n) $-2y(y+8)$

(o) $-x^2(x+6)$

(p) $3x^2(5x-2)$

3.3 Expanding and Simplifying Polynomial Expressions

Example 1 2(3x – 7) – 4(2x + 5) =6x – 14 – 8x – 40 = 6x – 8x –14 – 40 = -2x -54	*Multiply using the distributive property to remove the brackets. Remember that –4 changes the sign of both terms upon multiplication.* *Collect like terms. When moving terms, the sign in front of the term travels with the term.* *Simplify by adding/subtracting the like terms.*
Example 2 $5x(3 - x) - 2(x^2 - 4)$ $= 15x - 5x^2 - 2x^2 + 8$ $= -5x^2 - 2x^2 + 15x + 8$ $= -7x^2 + 15x + 8$	 *Expand.* *Collect like terms.* *Simplify.*

<u>Exercise 3.3:</u>

(a) $(x+3) - 2(x-2)$

(b) $5(x+8) + 4(x-2)$

(c) $x(3-x) + x(x+1)$

(d) $4\left(2x^2 + 6x - 4\right) - 3x\left(5x + 1\right)$

(e) $2(3x^2+4)+4(x^2-5)$

(f) $3x(2x+1) - x(4x-2)$

(g) $-(2x^2+4x-1)+3x(x-5)$