



Ratios, Rates and Proportions

Suggested time: 75 min

What's important in this lesson:

You will learn the vocabulary and procedures related to ratios, rates and proportions. You will apply ratios, rates and proportions to solve problems.

Complete these steps:

1. Read the lesson portion of the package on your own.
2. Complete the exercises as they appear in the lesson.
3. Check your answers with the answer key that your teacher has.
4. Ask for help at any point during the lesson.
5. Complete the "Unit 2, Lesson 1 Ratios, Rates and Proportions Assignment" and submit to your teacher for evaluation.

Hand-in the following to your teacher:

1. "Unit 2, Lesson 1 Ratios, Rates and Proportions Assignment"

Questions for the teacher:

Diagnostic/Introductory Activity:
Unit 2 Lesson 1



Using any tools or manipulatives provided by your teacher, demonstrate/draw as many representations of the following fraction. Show all of your ideas on this page.

$$\frac{3}{4}$$



Ratios, Rates and Proportions

Ratios and Rates

A ratio is a comparison of two or more quantities with the same units.

How can you write the ratio 4 to 8 mathematically?

1. Write it with words.	4 to 8
2. Write it with a colon.	4:8
3. Write it so it looks like a fraction.	$\frac{4}{8}$
4. Simplify. This is only possible if the quantities are both divisible by the same number.	$4 \text{ to } 8 = 1 \text{ to } 2$ $4:8 = 1:2$ $\frac{4}{8} = \frac{1}{2}$

Example:

In a pack of 9 gums, 3 are cherry flavour. Write each ratio in simplest terms.

- I. cherry flavour to all gums
- II. all gums to cherry flavoured gums
- III. cherry flavoured gums to other flavours

Solution:

- I. Ratio of cherry flavoured gums to all gums is 3 to 9 and can be written as 3:9.

Cherry:All = 3:9

To simplify a ratio, divide each term by the greatest common factor,
Therefore $3:9 = 1:3$.

In simplest form, Cherry:All = 1:3

- II. Ratio of all gums to cherry ones: 9 to 3 and in simplest form: 3 to 1

All: Cherry = 9:3

In simplest form, All:Cherry = 3:1

- III. Ratio of cherry flavoured to other flavoured gums
There are 9 gums, 3 are cherry; so $9 - 3 = 6$ are other flavours.

Cherry: Other flavours = 3:6



Exercise 1.1: Ratios

1. Write each ratio in two ways. Write your answer in simplest form.

<p>(a)</p> <p>triangles to total</p>	<p>(b)</p> <p>triangles to squares</p>
<p>(c)</p> <p>circles to squares</p>	<p>(d)</p> <p>total to circle</p>

2. Write each ratio in simplest form.

(a) 16:8	(b) $\frac{42}{39}$	(c) $\frac{15}{10}$	(d) 24:56	(e) $\frac{60}{66}$
(f) 20 to 25	(g) 27:18	(h) $\frac{13}{26}$	(i) 14 to 2	(j) $\frac{12}{14}$
(k) 28:32	(l) 4 to 20	(m) $\frac{50}{25}$	(n) 24:40	(o) 90 to 100

Student Handout: Unit 2 Lesson 1




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A rate is a comparison of two quantities with different units.

A unit rate is a comparison of two quantities in which the second term is 1.

The speed 65km/h is a rate. The distance in kilometers is compared to the time in hours.

Example:	Solution:
<p>Joanne earns \$78.20 in 8.5 hours. (\$78.20 in 8.5 hours is a RATE) What is Joanne's pay per hour?</p> 	<p>Joanne's hourly rate is</p> $\frac{\text{earnings}}{\text{time}} = \frac{78.20}{8.5}$ <p>= 9.2 ∴ Joanne's pay per hour is \$9.20/h (\$9.20/h is a UNIT RATE)</p>

Exercise 1.2 :

1. Find the unit rate.

Questions:	Solutions:
(a) He drove 202 kilometres in 2 hours	He drove _____ in 1 hour.
(b) The car went 176 meters in 2 seconds	The car went _____m in 1 second.
An international phone call costs \$15.69 for 46 minutes _____ cents per minute	The phone call costs _____ cents/min.

2. Find the unit rate.

(a) 128 seats in 8 rows	(b) 378 pages in 3 days
(c) 68 meters in 2 seconds	(d) 3,896 miles in 8 hours



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3. Circle the better buy. Show your work. Explain why it is the better buy.

(a)
 \$19 for 38 litres

$$\frac{\$19}{38L} = \$0.50 / L$$

This is the better buy – less money per litre

\$11.88 for 22 litres
 $\frac{\$11.88}{22L} = \$0.54 / L$

(b)
 \$1.82 for 7 litres

\$14 for 35 litres

(c)
 6 kilograms for \$2.58

27 kilograms for \$14.85



(d)

\$3.28 for 8 pounds

\$11.52 for 32 pounds

(e)

\$5.00 for 3 bags of chips

\$3.59 for 2 bags of chips

(f)

\$2.49 for 2000 mL of cola

\$0.99 for 500 mL of cola

(g)

\$6.79 for 3 bars of soap

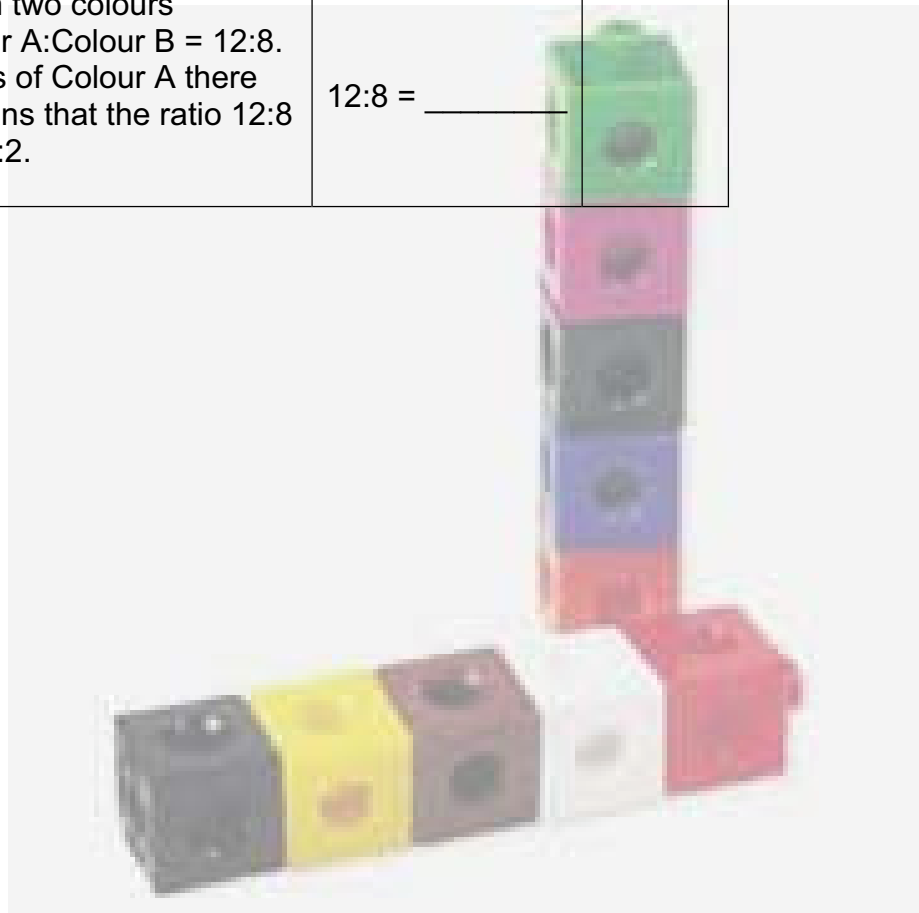
\$14.99 for 8 bars of soap



Using Ratio to Describe Composition of the Whole (using cube-a-links)

As you complete each problem, call the teacher over to check your solution. Ensure that your learning partner and you both agree with the solution.

Problem	Answer	Teacher initials
1. Select 25 cubes in only 2 colours. Use them to build anything you like. Write the ratio that describes the number of each colour used.		
2. Select a total of 12 cubes in two colours according to the ratio Colour A:Colour B = 2:1. Write the ratio for the colours you selected. Use them to build anything that you like.		
3. Select a total of 14 cubes in two colours according to the ratio Colour A:Colour B = 2:12. For every 1 cube of Colour A there are _____ cubes of Colour B. This means that 2:12 = _____. This is called an equivalent ratio. (Some discussion may occur here.)	2:12 = _____	
4. Select a total of 20 cubes in two colours according to the ratio Colour A:Colour B = 12:8. Show that for every 3 cubes of Colour A there are 2 of Colour B. This means that the ratio 12:8 is “equivalent” to the ratio 3:2.	12:8 = _____	





Solving Proportions

Suppose you make \$625 for working 5 weeks in a book shop. At this rate of pay how much money will you make in 10 weeks?

We can write a proportion in which the rates compare the amount of pay to the time worked.

$$\begin{array}{ccc} \text{Pay} & \longleftarrow & \text{Pay} \\ \text{time} & \longleftarrow & \text{time} \end{array} \quad \frac{625}{5} = \frac{x}{10}$$

Solution 1:	Solution 2:	Solution 3:
<p>We can determine the rate of pay for one week</p> $\frac{625}{5} = 125$ <p>Then we can use the weekly rate to calculate the pay for 10 weeks by multiplying.</p> $125 \times 10 = 1250$ <p>\therefore the pay for 10 weeks is \$1250</p>	<p>We can find x by determining the equivalent fraction.</p> $\frac{625}{5} = \frac{x}{10}$ <p>As we move from left side to right side, the denominator is multiplied by 2. If the proportion is true, then we must multiply the numerator on the left by 2 to get x.</p> $\frac{625(\times 2)}{5(\times 2)} = \frac{1250}{10}$ <p>\therefore the pay for 10 weeks is \$1250</p>	<p>Now that you understand how to find x, Here's a trick...</p> $\frac{625}{5} = \frac{x}{10}$ <p>we can cross multiply</p> $\frac{625}{5} = \frac{x}{10}$ <p>and then we can write:</p> $5 \times x = 625 \times 10$ $5x = 6250$ $\frac{5x}{5} = \frac{6250}{5}$ $x = 1250$ <p>\therefore the pay for 10 weeks is \$1250</p>

**Example:**

It is believed that there are 2 cats for every 1 dog. If there are 1236 dogs in the city, how many cats are there?

Solution:

<p>Cats to Dogs = 2:1</p> $\frac{\text{cats}}{\text{dogs}} = \frac{2}{1}$ $\frac{2}{1} = \frac{x}{1236}$ $1 \times x = 2 \times 1236$ $x = 2472$ <p>Therefore there are 2472 cats in the city.</p>	<p>We can write as a proportion and let x be the number of cats.</p> <p>Cross multiply</p> <p>Determine x.</p>
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Example:

Saul and his family have crossed the border to go out for dinner. The waiter said that they would accept \$80 Canadian for the \$60 US bill.

- What exchange rate did the restaurant offer?
- How many Canadian dollars would cover a \$100 US restaurant bill?

Solution:

- Find the restaurant's exchange rate.

$$\frac{\text{Canadian\$}}{\text{US\$}}$$

$$\rightarrow \frac{x}{\$1\text{US}} = \frac{80}{60}$$

$$60x = 80$$

$$x = \frac{80}{60}$$

$$x = 1.\dot{3} \cong 1.33$$

Therefore the exchange rate offered by the restaurant is \$1.33 Canadian for \$1 US

- Find the equivalent Canadian amount for a \$100 US bill.

Let x be the amount of Canadian \$

We can use the exchange rate from above.



Canadian\$

US\$

$$\rightarrow \frac{1.33}{\$1US} = \frac{x}{100}$$

$$x = 1.33(100)$$

$$x = 133$$



To pay the \$100 US bill, it will cost \$133 Canadian dollars.



Exercise 1.3: Solve for the unknown. Show your work.

Questions:	Solutions:
(a) $\frac{9}{4} = \frac{45}{x}$	
(b) $\frac{18}{20} = \frac{x}{10}$	
(c) $\frac{1}{2} = \frac{9}{x}$	
(d) $\frac{35}{20} = \frac{7}{x}$	
(e) $\frac{3}{x} = \frac{4}{36}$	
(f) $\frac{x}{3} = \frac{10}{5}$	
(g) $\frac{5}{4} = \frac{x}{12}$	



Applications

(a) A motorcycle engine burns a mixture of oil and gasoline in the ratio of 1:20, meaning 1 part of oil is mixed with 20 parts of gasoline. How much oil should be added to 7 L of gasoline?

(b) If it were possible to drain all the water from a 90 kg man, his dehydrated body would have a mass of 40 kg. Using this ratio, what would the mass of tissue be for a 117 kg man?

(c) The ratio of boys to girls in Grade 10 at Escarpment High is 7:6. There are 192 girls in the grade. How many boys are there?

(d) On a motorbike trip, Jen traveled 360 km. in 5 hours. At that rate, how far could she travel in 8 hours?

(e) On a map, 1.5 cm represents 13 km. What is the approximate distance between Toronto and Hamilton if the distance between them on the map is 31.2 cm?



Ratios, Rates and Proportions Assignment

1. Determine the following unit rates.

He made 27 calls in 9 hours.	
There were 81 seats in 9 rows.	
It flew 790 metres in 10 seconds.	
A 3.6 kg bag of cherries costs \$3.49.	
The sale sign indicated "4 for \$6.40".	

2. Solve for the variable in the following proportions.

$\frac{8}{n} = \frac{40}{35}$	
$\frac{7}{6} = \frac{35}{n}$	
$\frac{x}{6} = \frac{11}{2}$	
136 pages in 1 day = _____ pages in 2 days	
90 metres in 2 seconds = _____ metres in 4 seconds	

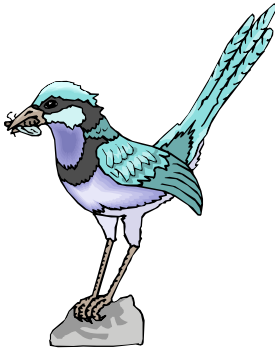


3. Complete the following.

- (a) It was recently estimated by the Extraterrestrial Alien Monitoring Agency, that unregistered extraterrestrials outnumber registered ones by about six-to-three. If there are 972 unregistered extraterrestrials, about how many of them are registered?



- (b) A scientist counted birds in a cornfield. He counted ten crows, six jays and twelve hawks. What was the ratio of jays to hawks?



- (c) Christina practices figure skating an average of 15 hours per week. Kayla practices figure skating an average of 1.5 hours for every hour Christina practices. How many hours a week does Kayla practice?