Student Instruction Sheet: Unit 4 Lesson 5

Optimization

Suggested time: 75 minutes

What's important in this lesson:
In this lesson you will apply your knowledge of table of values and graphs to solve maximization problems involving rectangles.

Complete these steps:

1. Read through the lesson portion of the package independently.
2. Complete any of the examples in the lesson.
3. Check your lesson answers with the lesson key your teacher has.
4. Seek assistance from the teacher as needed.
5. Complete the Assessment and Evaluation and submit for evaluation. Be sure to ask for any assistance when experiencing difficulties.

Hand-in the following to your teacher:

1. Assessment and Evaluation

Questions for the teacher:

Optimization

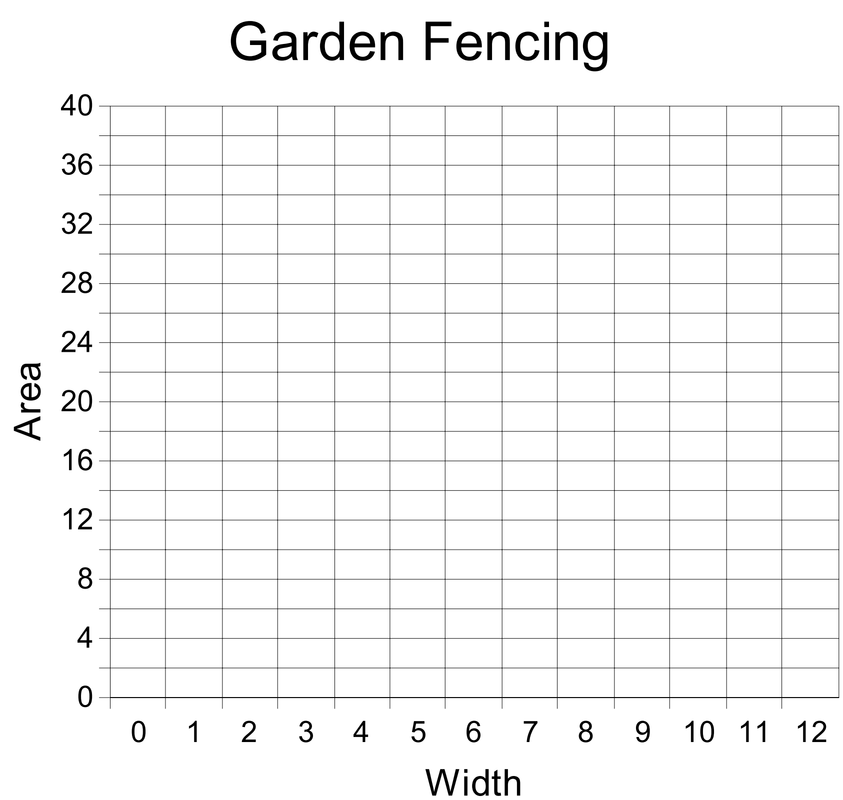
Example #1

Your neighbour has asked for your advice about building his garden. He wants to fence the largest rectangular garden possible with 24 metres of fencing. What do you think the largest rectangular garden will look like?

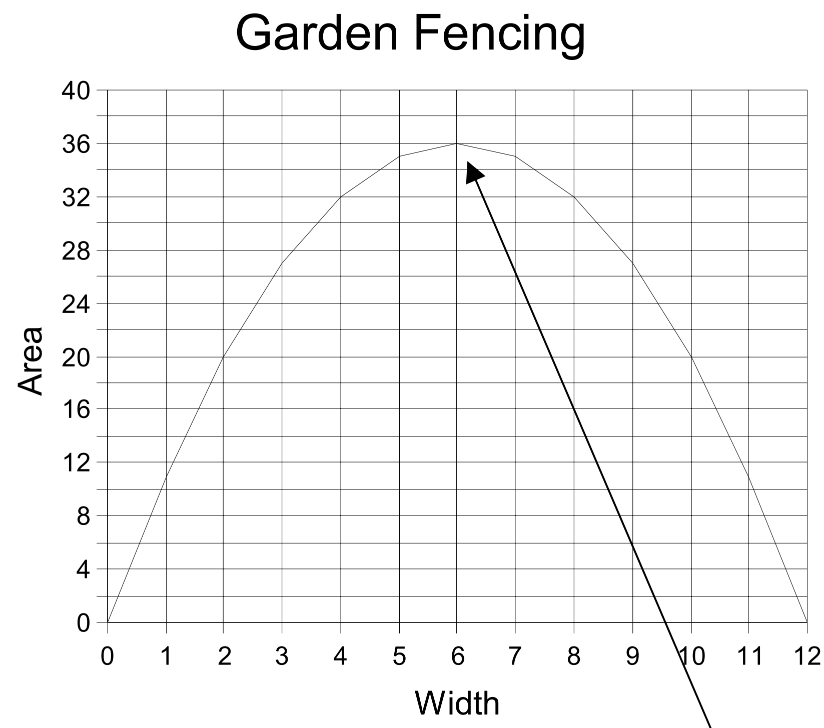
In order to solve the problem you can make use of your knowledge of tables of values and graphs.

Complete the table and graph.

Perimeter (m)	Width (m)	Length (m)	Area (m^2) $l \times w$
24	0	12	0
24	1	11	11
24	2	10	20
24	3		
24	4		
24	5		
24	6		
24	7		
24	8		
24	9		
24	10		
24	11		



Your graph should like the one below.



Notice the graph does not form a line. This is a non-linear relationship. As the width increase the area increase to a maximum point and then starts to decrease. You can use the graph to find the maximum area. The maximum area you can produce is the highest point on the graph. The point is (6, 36). In other words, when the width is 6m the area is $36m^2$.

Conclusion

Your neighbour can maximize the area of the garden by making the dimensions 6 x 6. These dimensions will produce a total area of $36m^2$

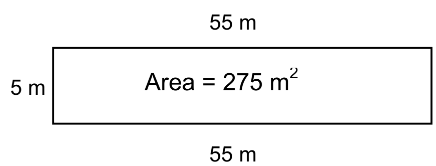
Optimization problems are a great application of when to use table of values and graphs to assist you in problem solving.

Example 2

The town planners have hired you to design a rectangular ice rink for the local park. They will provide 120 metres of fencing. Your design should enclose the greatest possible area for the skaters.

Explore

It is possible to build a long, narrow ice rink, as shown.



Area = length × width
Area = 5 × 55
Area = 275 m^2

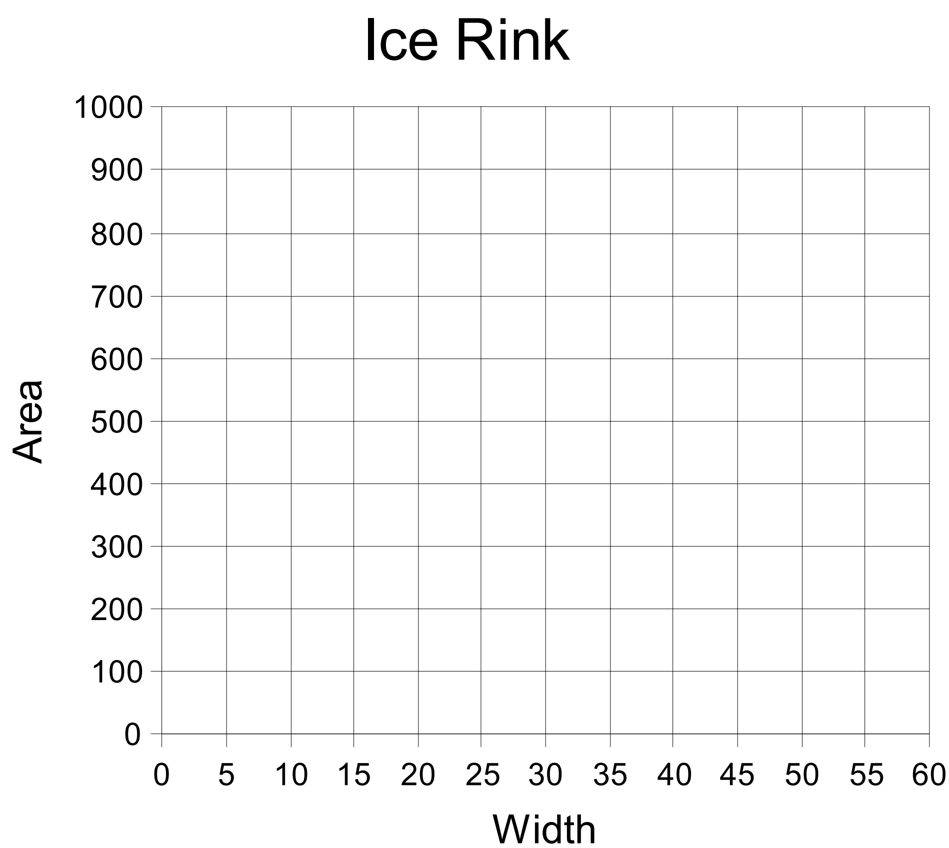
On a separate piece of paper, sketch *three* more ice rinks that have a larger area than this ice rink. Label the dimensions on the sketch and calculate the area.

1. Complete the table with all possible combinations of width and length for the ice rinks.

Perimeter (m)	Width (m)	Length (m)	Area (m^2) $l \times w$
120	0	60	0
120	5	55	275
120	10		
120			
120			
120			
120			
120			
120			
120			
120			
120			
120			

2. Describe what happens to the area when the width of the ice rink increases.

3. Construct a scatter plot of Area vs. Width.



4. Circle the region on the scatter plot where the area of the rink is the largest.

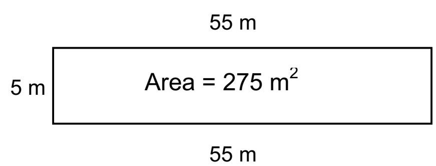
Conclusion

Write a report to the town advising them of the dimensions that would be best for the new ice rink. Justify your recommendation. Include a sketch and the area of the ice rink that you are recommending.

Example 2 - Solutions

The town planners have hired you to design a rectangular ice rink for the local park. They will provide 120 metres of fencing. Your design should enclose the greatest possible area for the skaters.

Explore
It is possible to build a long, narrow ice rink, as shown.



Area = length × width
Area = 5 × 55
Area = 275 m^2

On the back of this page, sketch *three* more ice rinks that have a larger area than this ice rink. Label the dimensions on the sketch and calculate the area.

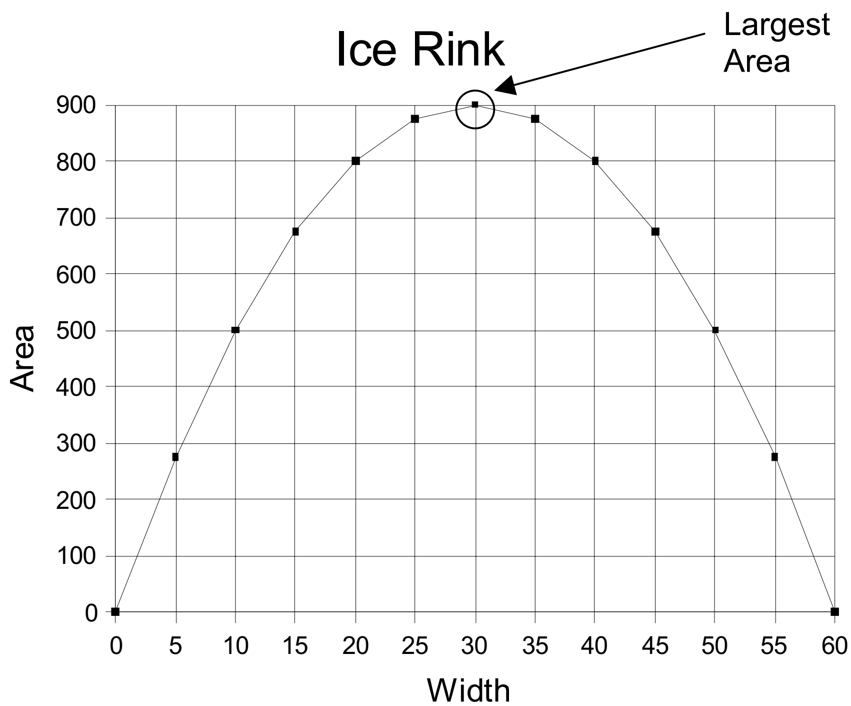
1. Complete the table with all possible combinations of width and length for the ice rinks.

Perimeter (m)	**Width (m)**	**Length (m)**	**Area (m^2)** ***l × w***
120	0	60	0
120	5	55	275
120	10	**50**	**500**
120	**15**	**45**	**675**
120	**20**	**40**	**800**
120	**25**	**35**	**875**
120	**30**	**30**	**900**
120	**35**	**25**	**875**
120	**40**	**20**	**800**
120	**45**	**15**	**675**
120	**50**	**10**	**500**
120	**55**	**5**	**275**
120	**60**	**0**	**0**

2. Describe what happens to the area when the width of the ice rink increases.

   3. As the width of the ice rink increase the area increases until it reaches a peak of 900m^2 when the width is 30. From this point the area decreases. Note that the increments are symmetrical.

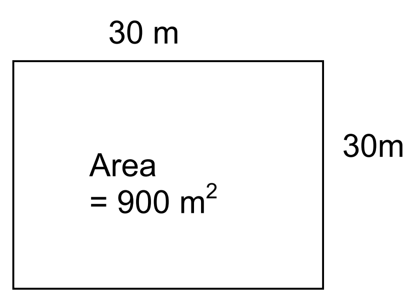
4. Construct a scatter plot of Area vs. Width.



5. Circle the region on the scatter plot where the area of the rink is the largest.

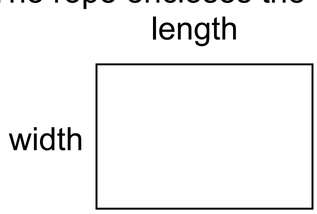
Conclusion
Write a report to the town advising them of the dimensions that would be best for the new ice rink. Justify your recommendation. Include a sketch and the area of the ice rink that you are recommending.

In order to create an ice rink with the largest possible area the town should build a square. The square rink will measure 30m by 30 m. The town may consider another size rink that will allow the skaters to have a longer length to skate.



Exercise.
A storeowner wants to create a rectangular area for a store display. He has 8 m of rope. What are the dimensions of the largest area he can enclose in each situation. Explain your answer.

(A) The rope encloses the entire area.

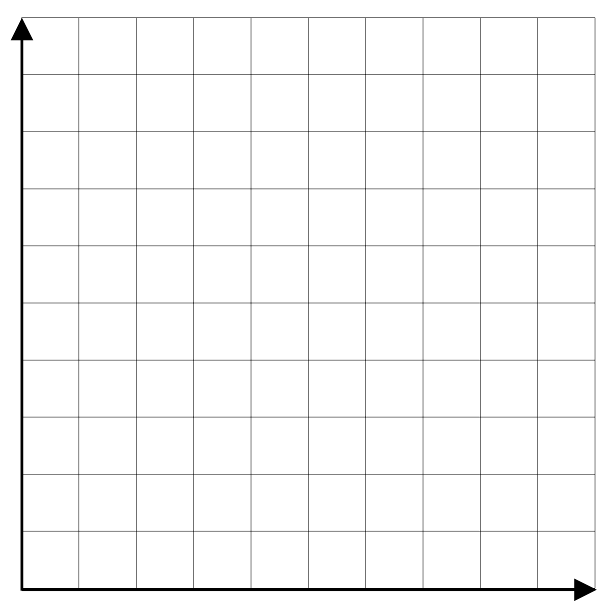


Complete the table with all possible combinations of width and length.

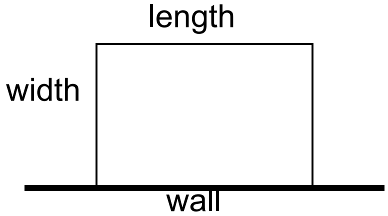
Perimeter (m)	**Width (m)**	**Length (m)**	**Area (m^2)** $l \times w$
8	0	4	
8	1	3	
8	2		
8	3		
8	4		

(a) Describe what happens to the area when the width of the display increases.

(b) Construct a scatter plot of area vs. width.



(B) There is a wall on one side.
(Note: the rope will be used for 2 widths and **1 length only**)

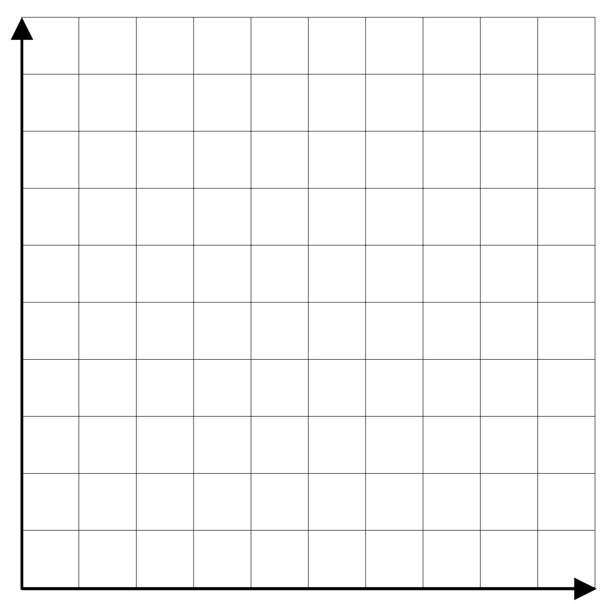


Complete the table with all possible combinations of width and length.

Perimeter (m)	**Width (m)**	**Length (m)**	**Area (m^2)** ***l* × *w***
8	0	8	
8	1	6	
8	2		
8	3		
8	4		

(a) Describe what happens to the area when the width of the display increases.

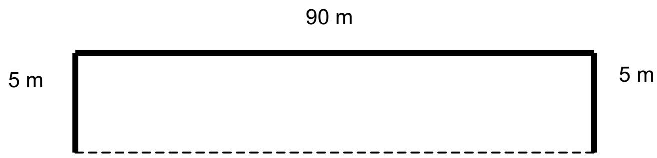
(b) Construct a scatter plot of area vs. width.



The city planners would also like you to design a swimming area at a local beach. There is 100 m of rope available to enclose the swimming area. The shore will be one side of the swimming area; so only three sides of the rectangle will be roped off. It is your job to design the largest rectangular swimming area.

Explore

It is possible to build a long, narrow swimming area.



Area = length $\times$ width
Area = 5×90
Area = 450 m^2

On a separate piece of paper, sketch three more swimming areas that have a larger area than this swimming area.

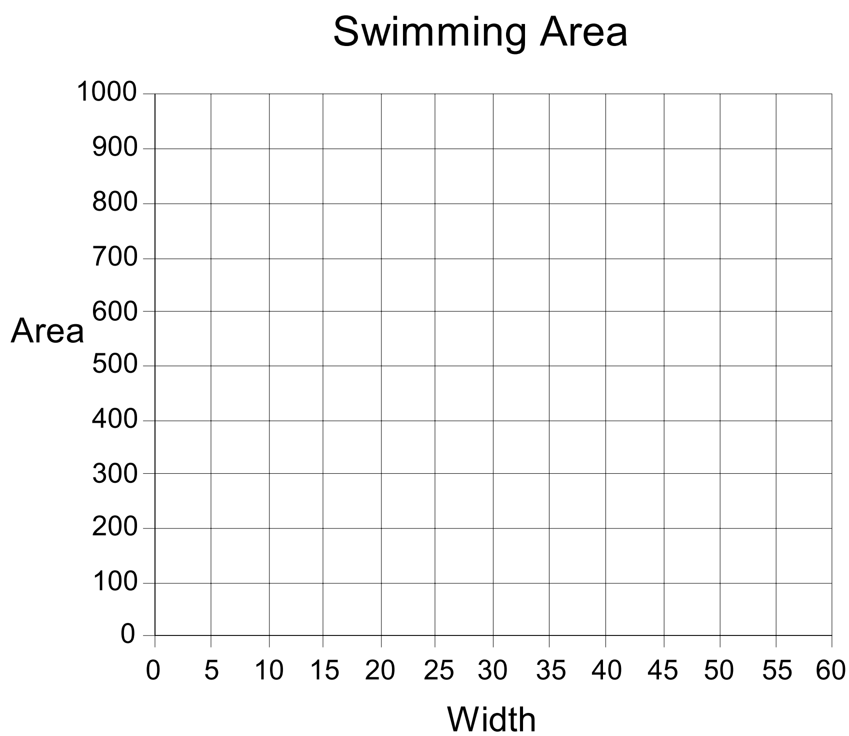
Label the dimensions on the sketch and calculate the area as shown above.

1. Complete the table with all possible combinations of width and length for the swimming area.

Perimeter (m)	Width (m)	Length (m)	Area (m^2) $l \times w$
100	0	100	0
100	5	90	450
100	10		
100			
100			
100			
100			
100			
100			
100			
100			

2. Describe what happens to the area when the width of the swimming area increases.

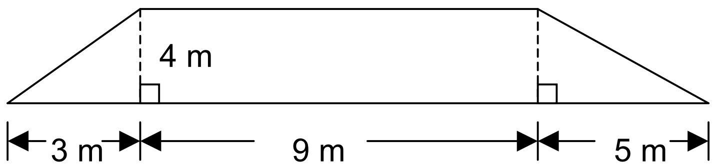
3. Construct a scatter plot of Area vs. Width.



4. Circle the region on the scatter plot where the swimming area is the largest.

5. Write a report to the town advising them of the dimensions that would be best for the new swimming area. Justify your choice. Include a sketch and the area of the swimming area that you are recommending.

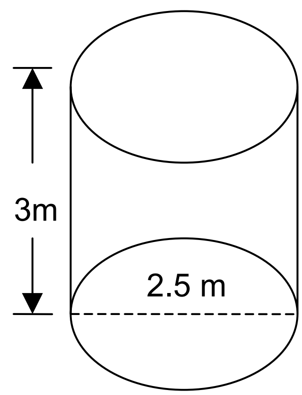
1. Calculate the perimeter and area of the figure correct to one decimal place.



{K 6}

2. Jordan uses oil to heat his house. An oil tank is used to store the oil before it is burned. Round all answers to one decimal place.

   (a) Calculate the volume of the oil tank.



{K 2}

   (b) If the oil tank is filled 30% full, then how much liquid is in the tank?

{A 2}

(c) Oil cost $ 50 per m^3. How much would it cost to fill the rest of the tank?

{Tips 3}

(d) Calculate the final price of the top up with taxes.

{A 2}

3. A farmer wishes to make a pen for his pigs. He has 200m of fencing available. What are the dimensions that will optimize the area?

   {A 5}