



Student Instruction Sheet: Unit 1, Lesson 2

Binomial Expressions

Suggested time: 75 minutes

What's important in this lesson:

In this lesson, you will learn how to expand and simplify binomial expressions.

Complete the following steps:

1. Read through the lesson portion of the package on your own.
2. Complete the exercises.
3. Check your answers with the Answer Key that your teacher has.
4. Seek assistance as needed.

Hand in the following:

1. Student Handout

Questions for the teacher:



Student Handout: Unit 1, Lesson 2

Vocabulary

A **variable** is a letter.

It represents the unknown value (a number).

A **coefficient** is the number in front of the variable(s).

In $2x$, 2 is the coefficient. In $9x^2$, 9 is the coefficient.

If the coefficient is 1, it is usually not written $1abc = abc$.

A **monomial** or **term** is an expression that involves multiplication of a *coefficient* and one or more *variables*.

$3x^2$ is a *monomial*.

xyz is a *monomial*.

7 is also a *monomial*, but because it doesn't have a variable, it is usually called a *constant*.

A **binomial** is the sum or difference of **two** monomials (terms). We will always see a plus or minus sign between the two terms.

$3x^2 + 6x$ is a *binomial*.

$5x - 7$ is a *binomial*.

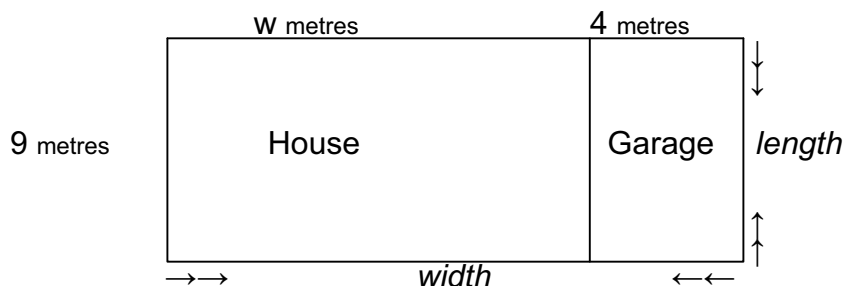
A **trinomial** is an expression formed by adding or subtracting **three** terms. $x^2 + 7x + 10$ is a *trinomial*.



Student Handout: Unit 1, Lesson 2

Topic 1: A Binomial Multiplied by a Monomial

Developers have decided that every house in a certain subdivision must have a length of 9 metres. Every house must also have a single-car garage attached at the side of the house. The garage must be 4 metres wide. However, the width of the houses may vary. Let the variable, w , represent the width of such a house.



Write the expressions that will find the Area of the house, and the Area of the garage. The measuring units are in metres, so the Area units are in square metres.

$$\text{Area} = \text{length} \times \text{width}$$

$$\begin{aligned} \text{Area house part} &= 9 \times w \\ &= 9w \text{ square metres} \end{aligned}$$

$$\begin{aligned} \text{Area garage part} &= 9 \times 4 \\ &= 36 \text{ square metres} \end{aligned}$$

To find the **Total Area** (house and garage together), write algebraically (leaving out the units) as

$$\begin{aligned} \text{Total Area} &= \text{length} \times \text{width all the way across} \\ &= 9 \times (w + 4) \\ &= 9(w + 4) \\ &= 9w + 36 \end{aligned}$$

which works out to the sum of the individual parts.

Notice that when we multiply a monomial and a binomial, $9(w + 4)$, we have to multiply **BOTH** parts of the binomial (in the brackets) by the monomial. In an earlier grade, you may have seen this idea. It's called the **Distributive Property**.

The mathematical word that tells us to multiply two expressions is "**expand**." A question such as "Expand $9(w + 4)$ " is asking us to multiply the first term by each term in the brackets. The answer, $9w + 36$, is **the expansion of** $9(w + 4)$.

Examples Expand each product. (Pay close attention to variables and exponents!)

$$\begin{aligned} &2(x + 7) \\ &= 2x + 14 \end{aligned}$$

$$\begin{aligned} &3k(2k - 8) \\ &= 6k^2 - 24k \end{aligned}$$

$$\begin{aligned} &4y(y + 9) \\ &= 4y^2 + 36y \end{aligned}$$

$$\begin{aligned} &5(x^2 + 3x - 12) \\ &= 5x^2 + 15x - 60 \end{aligned}$$



Student Handout: Unit 1, Lesson 2

Try the following questions: Expand each product.

[a] $5(2x + 14)$

=

[b] $-3(x^2 + 4x - 6)$

=

[c] $4x(7x + 1)$

=

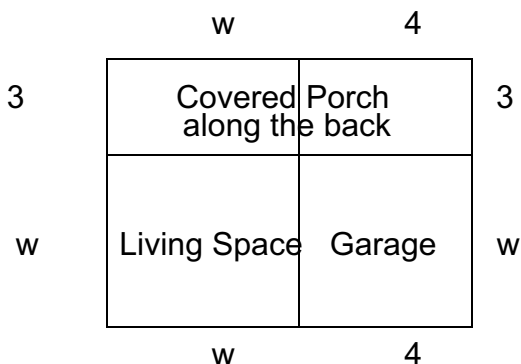
[d] $8m(m - 3)$

=

Topic 2: A Binomial Multiplied by a Binomial

Imagine a different subdivision where all of the houses have a square living space with a 4 m-wide garage on the side and a covered porch 3m in length, on the back. Buyers are allowed to choose the measurement of the living space.

Let the variable **w** represent both width and length of the square living space, since these measurements are equal. What is the expression for the **Total Area** of the living space, covered porch, and garage?



Area = length \times width.

Fill in the top left area with $3w$. Fill in the top right area with 12.

Fill in the bottom left area with w^2 . Fill in the bottom right area with $4w$.

For **Total Area**, you add them together, so you get $w^2 + 3w + 4w + 12 = w^2 + 7w + 12$.

Written algebraically, **Total Area** = length \times width = $(w + 3)(w + 4) = w^2 + 7w + 12$.

From the diagram, you can see that if you multiply a binomial by a binomial, you must do 4 separate multiplications and then combine like terms to get your final answer.



Student Handout: Unit 1, Lesson 2

Many teachers suggest that students use the word **FOIL** to help them remember what has to be multiplied.

F – (**First**) Multiply the first terms from each bracket.

O – (**Outside**) Multiply the first term in the first bracket by the second term in the second bracket. These are the terms furthest apart.

I – (**Inside**) Multiply the second term in the first bracket by the first term in the second bracket. These are the terms closest together.

L – (**Last**) Multiply the last terms from each bracket.

The diagrams work nicely if there are only plus signs in the binomials. However, with minus signs, it's easiest to just follow the pattern. **Remember the rules for multiplying and adding integers.**

Read the following examples *carefully* before doing the worksheet.

Expanding and Simplifying a Binomial Times a Binomial

Examples:

Expand and simplify each product of two binomials.

$$\begin{aligned}(m + 2)(m + 8) \\ &= m^2 + 8m + 2m + 16 \\ &= m^2 + 10m + 16\end{aligned}$$

$$\begin{aligned}(x + 7)(x - 5) \\ &= x^2 - 5x + 7x - 35 \\ &= x^2 + 2x - 35\end{aligned}$$

$$\begin{aligned}(x + 8)(x - 8) \\ &= x^2 - 8x + 8x - 64 \\ &= x^2 - 64\end{aligned}$$

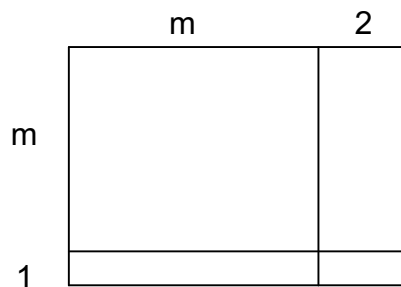
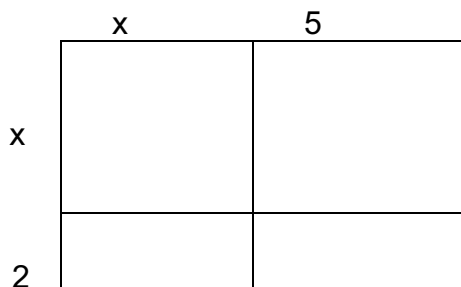
$$\begin{aligned}(k + 3)(k - 9) \\ &= k^2 - 9k + 3k - 27 \\ &= k^2 - 6k - 27\end{aligned}$$

$$\begin{aligned}(y - 4)(y - 12) \\ &= y^2 - 12y - 4y + 48 \\ &= y^2 - 16y + 48\end{aligned}$$

$$\begin{aligned}(2m - 3)(2m + 5) \\ &= 4m^2 + 10m - 6m - 15 \\ &= 4m^2 + 4m - 15\end{aligned}$$

Now try the following.

1. Label **each section** of each diagram with its area.





Student Handout: Unit 1, Lesson 2

2. Expand and simplify each product of two binomials.

[a] $(x + 4)(x + 9)$

[b] $(x + 11)(x - 3)$

[c] $(x - 7)(x + 7)$

[d] $(x - 7)(x + 6)$

[e] $(x - 2)(x - 10)$

[f] $(2m + 3)(2m - 3)$

Topic 3: Squaring a Binomial

An exponent tells us to **repeatedly multiply** whatever is just in front of that exponent. The exponent also tells us **how many times** to multiply whatever is in front of the exponent.

For example, $m^3 = (m)(m)(m)$
 $2m^3 = (2)(m)(m)(m)$
 $(2m)^3 = (2m)(2m)(2m)$

From the last example, we see that if there is an exponent right after a bracket, we must list (multiply) the entire bracket the number of times shown by the exponent.

If we see $(x + 7)^2$, the first step is to rewrite the expression in its long form. Then we can use the method from Topic 2 to expand and simplify. For example,

$$\begin{aligned}(x + 7)^2 &= (x + 7)(x + 7) \\ &= x^2 + 7x + 7x + 49 \\ &= x^2 + 14x + 49\end{aligned}$$

Notice what happens when there is a minus sign in the binomial.

$$\begin{aligned}(m - 5)^2 &= (m - 5)(m - 5) \\ &= m^2 - 5m - 5m + 25 \\ &= m^2 - 10m + 25\end{aligned}$$



Student Handout: Unit 1, Lesson 2

Try the following.

1. Expand and simplify.

[a] $(x + 8)^2$

[b] $(x - 6)^2$

[c] $(x + 1)^2$

[d] $(a + b)^2$

2. [a] Describe any patterns or relationships you see among the answers to Question 1.

Hint: To get the middle number in the answer, beside the x , what operation is done to the number in the binomial? _____ to itself

Hint: To get the last number of the answer, what operation is done to the number in the binomial? _____ by itself

[b] Based on the patterns you found in part [a] above, predict what binomial was squared to get each of the following trinomials.

1. $x^2 + 6x + 9 = (\quad)^2$

2. $x^2 - 6x + 9 = (\quad)^2$

3. $x^2 + 18x + 81 = (\quad)^2$

4. $x^2 - 20x + 100 = (\quad)^2$



Student Handout: Unit 1, Lesson 2

Topic 4: Combining All of the Skills Learned in Topics 1, 2, and 3

When an expression involves multiplication and powers, it is important to remember the **order of operations**: BEDMAS. Do operations in this order: Brackets inside, Exponents, Division/Multiplication, Addition/Subtraction.

1. If there is an exponent on a bracket, deal with that exponent first. (An exponent on a variable is not a problem unless we are asked to evaluate the expression.)
2. If you have a constant in front of two binomials, expand the product of the two binomials *first* before you multiply everything by the constant.
3. Add individual terms at the end of the expression last.

Examples: Expand and simplify.

$$\begin{aligned} \text{[a]} \quad & 2(x + 3)^2 - 8 \\ & = 2(x + 3)(x + 3) - 8 \\ & = 2(x^2 + 3x + 3x + 9) - 8 \end{aligned}$$

Drop the exponent and write out the bracket twice.

Use FOIL. Put the binomial product inside a new bracket because everything from the square of the original bracket still has to be multiplied by the constant 2.

$$\begin{aligned} & = 2x^2 + 6x + 6x + 18 - 8 \\ & = 2x^2 + 12x + 10 \end{aligned}$$

Multiply each term in the new bracket by 2.

Collect like terms.

$$\begin{aligned} \text{[b]} \quad & -4(m - 5)^2 + 2 \\ & = -4(m - 5)(m - 5) + 2 \\ & = -4(m^2 - 5m - 5m + 25) + 2 \\ & = -4m^2 + 20m + 20m - 100 + 2 \\ & = -4m^2 + 40m - 98 \end{aligned}$$

You try: Expand and simplify.

$$\text{[a]} \quad 3(x - 2)^2 - 5$$

$$\text{[b]} \quad -2(x + 7)^2 + 35$$