



## Student Instruction Sheet: Unit 4, Lesson 1

### Similar Triangles

Suggested Time: 75 minutes

#### What's important in this lesson:

In this lesson, you will learn how to solve similar triangles.

#### Complete the following steps:

1. Read through the lesson portion of the package on your own.
2. Complete all of the exercises.
3. Check your answers with the Answer Key that your teacher has.
4. Seek assistance from the teacher as needed.
5. Complete the Assessment and Evaluation and hand it in. Be sure to ask the teacher for assistance if you are having difficulty.

#### Hand in the following:

1. Student Handout
2. Assessment and Evaluation sheet

#### Questions for the teacher:



## Student Handout: Unit 4, Lesson 1

### Vocabulary about Similar Triangles

Two figures are **congruent** if they are exactly the same size and shape.

Two figures are **similar** if they are the same shape, but different sizes.

The symbol “ $\sim$ ” is a short form that means “is similar to.”

The word **corresponding** is used frequently in this lesson. The meaning might be obvious, but make sure that you understand how it is being used in this context. A simpler word for “corresponding” might be “matching.”

If we were comparing two triangles, **each triangle** would have a short side, a middle side, and a long side. The shortest side of each triangle would make a pair of **corresponding sides**. Similarly, the largest angle from each triangle would make a pair of **corresponding angles**.

If we were to take the ratio of the shortest side to the longest side within one of the triangles, we could also refer to the **corresponding ratio** in the other triangle.

A **proportion** is an equation that **relates two equivalent ratios**.

In this lesson, we will investigate exactly how to tell whether two figures are similar. This will involve looking at three things:

- the measurement of the angles in the figure
- the ratio between the lengths of corresponding sides in the two figures
- the ratios of pairs of corresponding sides within the figures



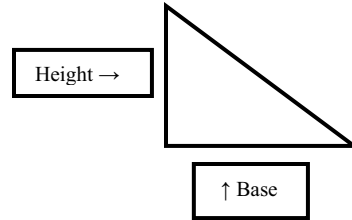
# Student Handout: Unit 4, Lesson 1



## Topic 1: Comparing Angles and Sides

1. Using the grid paper on the previous page, construct three **right-angle** triangles with the following dimensions:

Triangle	Base	Height
A	4 cm	3 cm
B	8 cm	6cm
C	12 cm	9 cm



Use a protractor and a ruler to measure the triangles you have drawn, and record the results in the chart below.

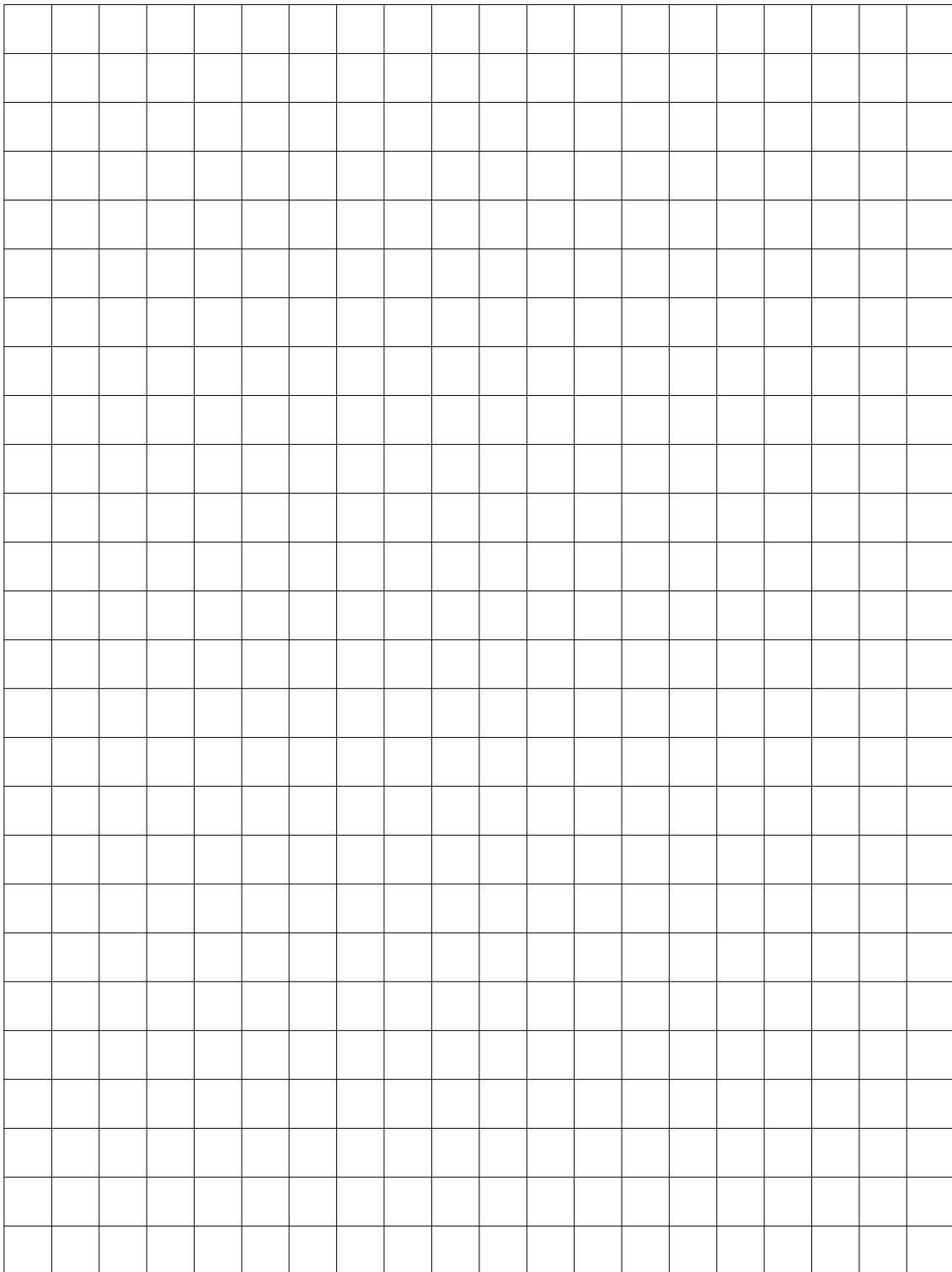
Triangle	Acute Angle on Base	Acute Angle at Top	Length of Hypotenuse
A			
B			
C			

[a] Describe the relationship between pairs of corresponding lengths in triangles A and B.

[b] Describe the relationship between pairs of corresponding lengths in triangles A and C.

[c] Describe the relationship among the corresponding angles in all three triangles.

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2. On grid paper on the previous page, mark a set of axes and draw a triangle with vertices at  $A(0,0)$ ,  $B(-2, 3)$ , and  $C(3, 4)$ . Then draw a second triangle with vertices at  $D(0,0)$ ,  $E(-4, 6)$ , and  $F(6, 8)$ .

Use a protractor and ruler to measure both triangles, and record the results in the chart below.

Triangle	Length of Left Side	Length of Right Side	Length of Top	Top Left Angle	Top Right Angle
$\triangle ABC$	(AB)	(AC)	(BC)	$\angle B$	$\angle C$
$\triangle DEF$	(DE)	(DF)	(FE)	$\angle E$	$\angle F$

[a] Explain why we didn't need to measure the bottom angle in each triangle.

[b] Describe the relationship between the side lengths for the two triangles.

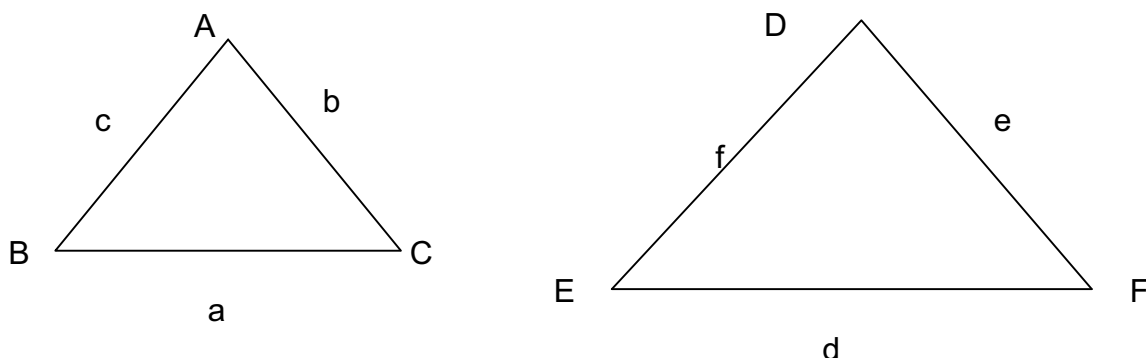
[c] Describe the relationship between the angles for the two triangles.



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### Topic 2: Properties of Similar Triangles

1. If two triangles are similar, then the corresponding **angles** must be equal.
2. The lengths of the corresponding **sides** make a proportion.



If the triangles above are **similar**, we could state the similarity by writing  $\triangle ABC \sim \triangle DEF$ , and the following statements would be true.

$$\begin{aligned}\angle A &= \angle D \\ \angle B &= \angle E \\ \angle C &= \angle F\end{aligned}$$

We can name sides, using either the two capital letters for the endpoints or the small letter that matches the opposite vertex. This means we could write the proportion that relates the side lengths in two ways.

$$\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \quad \text{or} \quad \frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

In these proportions, we set up the side lengths from **one triangle in the top line** and then put the corresponding side from the **other triangle underneath**.

It's important to realize that if **either** of the two properties can be shown to be true for a pair of triangles, then the triangles are similar and the other property must also be true. In other words, if you measure the angles of two triangles and see that all the corresponding angles are equal, then you don't have to check the side lengths because they will also be proportional. Similarly, you could measure the sides to see whether they are proportional, and if so, that would guarantee that the corresponding angles would also be equal.

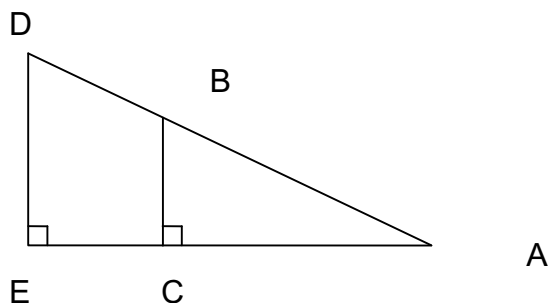


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### Topic 3: Solving for Length of Unknown Side

In mathematics, when we are asked to solve for the length of an unknown side, we often need to identify two similar triangles and then *set up and solve a proportion* to find the length of the requested side. There are a few different ways in which similar triangles are set up.

#### Type 1



In this situation, we have a right triangle nested inside a larger right triangle. From the properties of similar triangles, we know that if two triangles have all corresponding angles equal, they must be similar, and the corresponding sides must be proportional. In this case, we know that  $\triangle ABC \sim \triangle ADE$  because

$\angle A$  is common to both triangles.

$\angle C = \angle E$  because both are labelled as right angles.

$\angle B = \angle D$  **either** because the two vertical lines are parallel, **or** because all triangles have interior angles adding up to  $180^\circ$ , and as soon as the first two pairs are shown to be equal, the third pair will automatically be equal.

Once we are sure that the two triangles are similar, we can write the proportion relating the side lengths.

$$\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}$$

Even though this proportion can be used to relate all pairs of corresponding sides, we will often use only two of the fractions. We will usually have **three** side lengths available and only **one** side length requested, so we will use the two fractions that match the given information.

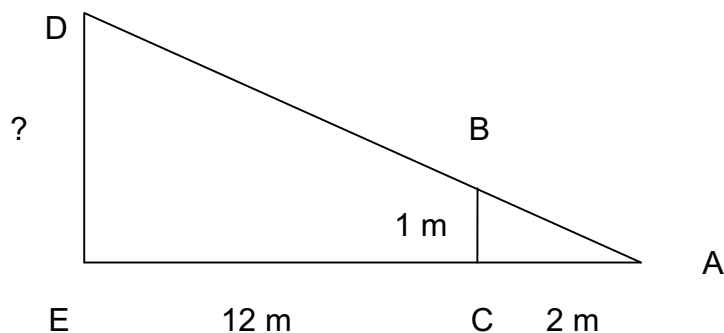




## Student Handout: Unit 4, Lesson 1

### Example 1

A student wants to estimate the height of a tree that has a clearly visible shadow. She places a metre stick so that the shadow of the metre stick lines up exactly with the end of the shadow of the tree. She then uses a long tape measure to measure the lengths of the shadows. She finds that the shadow of the metre stick is two metres long, and the distance from the metre stick to the tree is 12 metres.



We know that the triangles ABC and ADE are similar, so we can set up a proportion.

$$\frac{BC}{DE} = \frac{AC}{AE}$$

Now we substitute the given measurements to get

$$\frac{1}{DE} = \frac{2}{14}$$

*Notice that we had to add 12 + 2 to get the length of the base of the larger triangle!*

We solve the equation.

$$\begin{aligned}2 DE &= 14 \\ DE &= 14 \div 2 \\ DE &= 7\end{aligned}$$

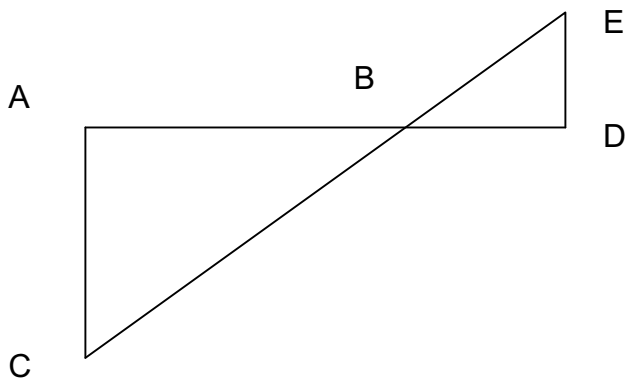
Therefore, the height of the tree is 7 metres.



## Student Handout: Unit 4, Lesson 1

### Topic 3: Solving for Length of Unknown Side, continued

#### Type 2



In a diagram of this type, we will be told or shown that the lines AC and DE are parallel. If these lines are parallel, then we know that  $\angle A = \angle D$  and  $\angle C = \angle E$ .

Because of the equal angle property, we can conclude that  $\triangle ABC \sim \triangle DBE$ .

Because of the similarity, we know that the sides must obey the proportion

$$\frac{AB}{DB} = \frac{AC}{DE} = \frac{BC}{BE}$$

#### Example 1

Assume, in the above diagram, that  $AB = 80$  m,  $BD = 4$  m, and  $DE = 3$  m. Solve for the length AC.

Substituting into the proportion, we get

$$\frac{80}{4} = \frac{AC}{3}$$

Solving the equation, we get

$$\begin{aligned} 4 AC &= 240 \\ AC &= 240 \div 4 \\ AC &= 60 \end{aligned}$$

Therefore, the requested length is 60 metres.



## Assessment and Evaluation: Unit 4, Lesson 1

1. If  $\triangle ABC \sim \triangle DEF$ , then

[a]  $\angle B = \angle$

[b]  $\frac{AB}{DE} = \frac{AC}{EF} = \frac{BC}{EF}$

2. Classify each statement as true or false.

\_\_\_\_\_ If two triangles have different angle measurements, then they can't be similar.

\_\_\_\_\_ A triangle with side lengths of 3 cm, 4 cm, and 5 cm must be similar to a triangle with side lengths of 6 cm, 8 cm, and 10 cm.

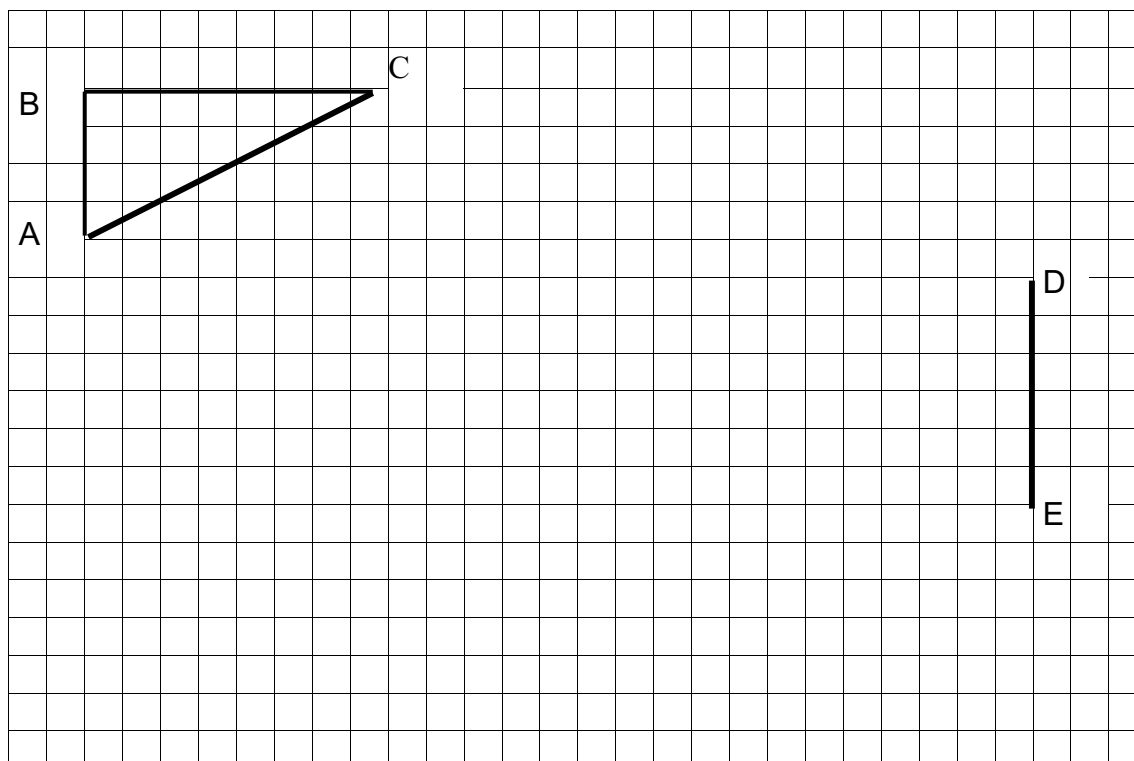
\_\_\_\_\_ All equilateral triangles are similar.

\_\_\_\_\_ All isosceles triangles are similar.

\_\_\_\_\_ If  $\triangle ABC \sim \triangle DEF$  **and**  $\triangle ABC \sim \triangle FED$  then  $\angle A = \angle C = \angle D = \angle F$ .

\_\_\_\_\_ All squares are similar.

3. Finish  $\triangle DEF$  so that  $\triangle DEF \sim \triangle ABC$ .





## Assessment and Evaluation: Unit 4, Lesson 1

4. A student is asked by her teacher to estimate the height of her apartment building.

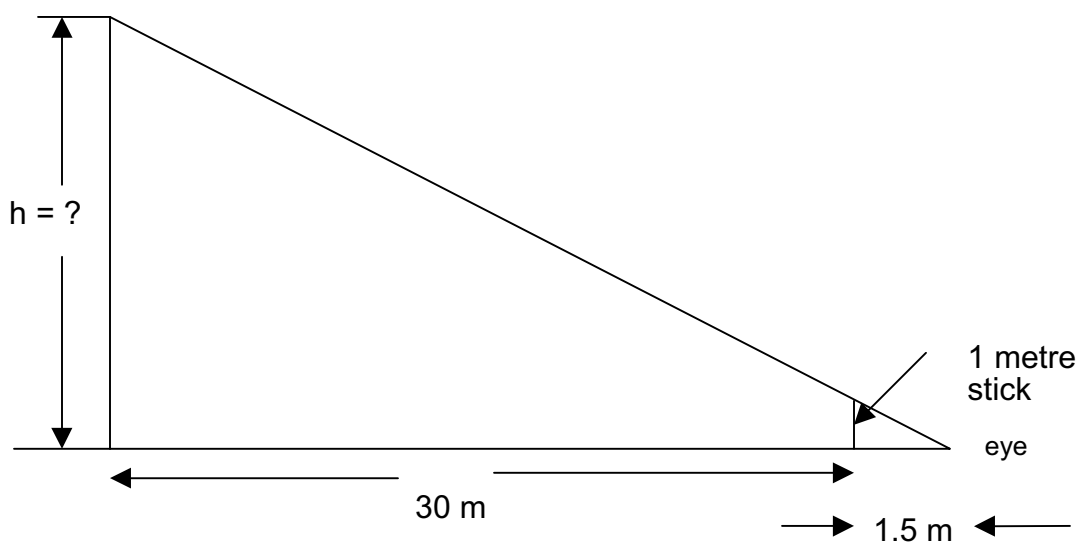
She goes to the parking lot beside her building with a metre stick and takes the following measurements:

The distance at ground level from the building to the upright metre stick is 30 m.

The distance from the base of the metre stick to eye position is 1.5 m.

The student can see the top of her building lined up with the tip of the metre stick.

Use similar triangles to solve for the height of the building.

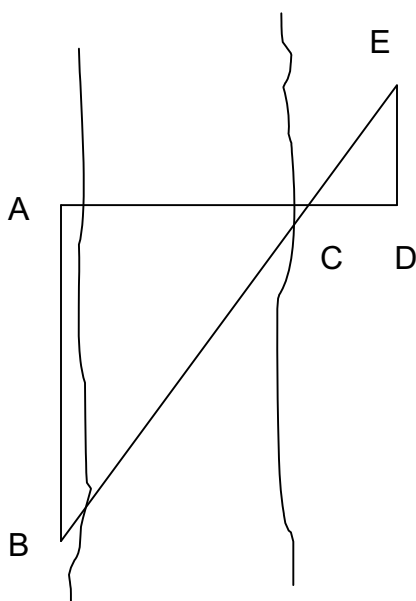




## Assessment and Evaluation: Unit 4, Lesson 1

5. A surveyor wants to determine the distance across a ravine. He has his assistant place two markers, which are 200 m apart, at points A and B on the far side of the ravine.

Next, he sets up a measured triangle on his side of the ravine, as shown. The diagram is a top view.



He measures Angle A and Angle D both at  $90^\circ$ .

The length of CD is 6 m and the length of DE is 12 m.

**Find the length of AC to get the width of the ravine.**