Student Instruction Sheet: Unit 4, Lesson 2

Ratios of Sides of Right-Angle Triangles

Suggested Time: 75 minutes

What's important in this lesson:

In this lesson, you will learn through investigation, the relationship between/among the sides of a right triangle and the ratios of sine, cosine, and tangent.

Complete the following steps:

1. Read through the lesson portion of the package on your own.
2. Complete all of the exercises.
3. Check your answers with the Answer Key that your teacher has.
4. Seek assistance from the teacher as needed.
5. Complete the Assessment and Evaluation and hand it in. Be sure to ask the teacher for assistance if you are having difficulty.

Hand in the following:

1. Student Handout
2. Assessment and Evaluation sheet

Questions for the teacher:

Student Handout: Unit 4, Lesson 2

Background

In the previous lesson, we worked with similar triangles. The focus of that lesson was on setting up ratios of sides between two similar triangles of a different size. In this lesson, we will establish definitions that will allow us to use the idea of similar triangles as it is applied to measured angles, as well as to measured sides.

Calculator Skills

The following lesson should help you understand exactly what the trigonometric ratios of an angle are. First, however, you should know how to get a value for a trigonometric ratio from your calculator.

Your calculator should have three buttons that are labelled **sin**, **cos**, and **tan**. If it doesn't have these buttons, you'll have to find a calculator that does!

Power up your calculator and hit the **sin** button. **If** you see the word "sin" appear in the display*, use the following instructions to get the sine ratio of a given angle.

- Press the "sin" key.
- Input the angle. (Your calculator should be set to degrees; get help from your teacher, if necessary.)
- Press the "= " key.

You should see a decimal value that is bigger than 0, but smaller than 1.

To verify, calculate sin 30°. You should get a value of 0.5 .

* If you got a "0" in the display after you hit the "sin" key, follow these instructions instead:

- Input the angle.
- Press the "sin" key.

You should see the decimal value right after you hit the "sin" key without having to hit the "=" key.

Student Handout: Unit 4, Lesson 2

Topic 1: Definitions

In any given right-angle triangle, there will always be two acute angles and a right angle. The acute angle in the question we will be looking at will be called the **reference angle**. You have to choose the reference angle! (For example, in the diagram below, you must choose either Angle A or Angle C.)

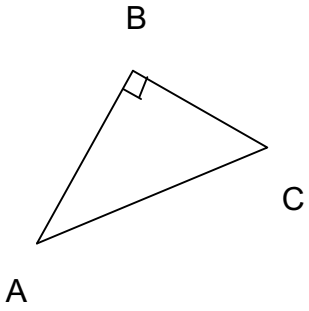
In any right-angle triangle, the side that is across from the right angle is called the **hypotenuse.** This is always the **longest side** in the right triangle.

The side that joins the hypotenuse to form the reference angle is called the **adjacent side** of the right triangle.

The side that is across from the reference angle and forms the right angle with the adjacent side is called the **opposite side** of the right triangle.

Because we can draw right triangles in any orientation and choose either of two acute angles to be our reference angle, the locations of the adjacent and opposite sides can't be predicted in terms of bottom, top, right, or left.

Always find the right angle first to label the hypotenuse. Then identify the reference angle to find the adjacent side. The opposite side will be the one left over.



If we choose $\angle$ A as our reference angle, then: AC is the hypotenuse
AB is the adjacent side
BC is the opposite side

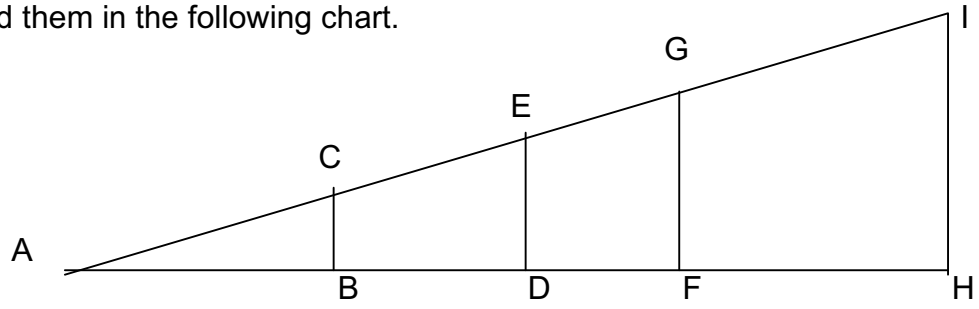
If we choose $\angle$ C as the reference angle instead, then:
AC is still the hypotenuse, but
BC will be the adjacent side, and
AB will be the opposite side

The sine, cosine, and tangent ratios of an angle are called **trigonometric** ratios. These ratios identify the ratios of sides within **any** right triangle for a given measured reference angle.

Student Handout: Unit 4, Lesson 2

Topic 2: Tan Ratio

For the diagram below, measure the requested lengths to the nearest mm and record them in the following chart.



Triangle	Length of Hypotenuse (mm)	Length of Side Adjacent to ∠A (mm)	Length of Side Opposite to ∠A	Measure of ∠A (°)
Δ ABC				
Δ ADE				
Δ AFG				
Δ AHI				

Now that we have recorded our lengths, we can compare the ratios of different pairs of sides.

Triangle	Opposite Side Length (divided by) Adjacent Side Length
Δ ABC	
Δ ADE	
Δ AFG	
Δ AHI	

Using a scientific calculator, find the value of the tangent ratio of the angle you measured for ∠A. How does this value compare to the values you have in your table?

Topic 3: Sine Ratio

Triangle	$\frac{\text{Opposite Side Length}}{\text{Hypotenuse Length}}$
∆ ABC	
∆ ADE	
∆ AFG	
∆ AHI	

Using a scientific calculator, find the value of the sine ratio of the angle you measured for ∠A. How does this value compare to the values you have in your table?

Triangle	$\frac{\text{Adjacent Side length}}{\text{Hypotenuse length}}$
∆ ABC	
∆ ADE	
∆ AFG	
∆ AHI	

Using a scientific calculator, find the value of the cosine ratio of the angle you measured for ∠A. How does this value compare to the values you have in your table?

The general idea is this: You can build a right triangle of any size, using a specified measurement for the acute reference angle, and the ratios between the sides will always be the same for that angle. These ratios can be obtained from your calculator, so you don't have to do the actual construction and measurement of the triangle.

Student Handout: Unit 4, Lesson 2

Topic 4: The Trig Ratios – Memorize them!

The following definitions apply to any right-angle triangle with a specified acute reference angle:

The sine ratio of the angle is found by dividing the length of the side opposite to the reference angle by the length of the hypotenuse.

$$\text{sine ratio} = \frac{\text{opposite}}{\text{hypotenuse}}$$

The cosine ratio of the angle is found by dividing the length of the side opposite to the reference angle by the length of the hypotenuse.

$$\text{cosine ratio} = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The tangent ratio of the angle is found by dividing the length of the side opposite to the reference angle by the length of the side adjacent to the reference angle.

$$\text{tangent ratio} = \frac{\text{opposite}}{\text{adjacent}}$$

For the sake of speed, we usually write these relationships as follows:

$$\sin = \frac{\text{opp}}{\text{hyp}} \qquad \cos = \frac{\text{adj}}{\text{hyp}} \qquad \tan = \frac{\text{opp}}{\text{adj}}$$

If we take the first letter from each word, we can create a nonsense word, SOHCAHTOA, which some people use as a memory aid. When we do word problems involving trigonometry, we should always start by deciding which of the three ratios we need for the problem we are being asked to do.

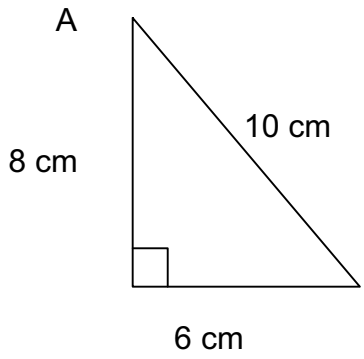
Assessment and Evaluation: Unit 4, Lesson 2

1. Use a scientific calculator to evaluate each of the following ratios.

[a] sin 40° = [b] cos 32° = [c] tan 65° =

[d] cos 4° = [e] tan 45° = [f] sin 76° =

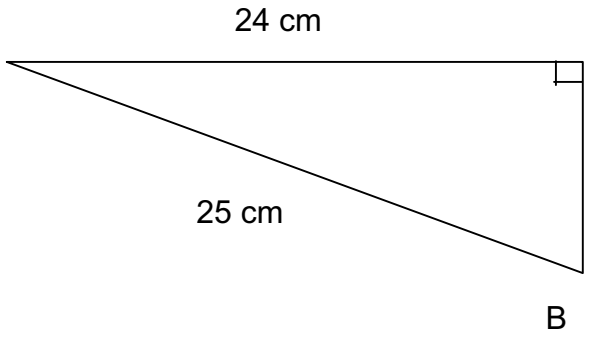
2. For the triangles below, identify the adjacent side, the opposite side, and the hypotenuse, and use your definitions for the ratios to find the values requested.



sin ∠ A =

cos ∠ A =

tan ∠ A =



sin ∠ B =

cos ∠ B =

tan ∠ B =