Solving Problems Using Trigonometric Ratios and the Pythagorean Theorem

Suggested Time: 75 minutes

What's important in this lesson:

In this lesson, you will solve problems using trigonometry (sin, cos, tan) and the Pythagorean Theorem ($a^2 + b^2 = c^2$).

Complete the following steps:

1. Read through the lesson portion of the package on your own.
2. Complete all of the exercises in the lesson.
3. Check your answers with the Answer Key that your teacher has.
4. Seek assistance from the teacher as needed.
5. Complete the Assessment and Evaluation and hand it in. Be sure to ask the teacher for assistance if you are having difficulty.

Hand in the following:

1. Student Handout
2. Assessment and Evaluation

Questions for the teacher:

Vocabulary for Trig Word Problems

An ***angle of elevation*** is an angle that is measured between the **horizontal base** of a right triangle and the hypotenuse. In a question of this type, the hypotenuse will often be an imaginary sight line. For example, if you are looking up at the top of a tower from the ground, the horizontal base of the triangle is measured from you to the base of the tower. The *angle of elevation* is the angle between the ground and the imaginary line between you and the top of the tower.

An ***angle of depression*** is an angle that is measured between an imaginary horizontal line drawn out from a high point and the hypotenuse of a right triangle joining the high point to some lower point.

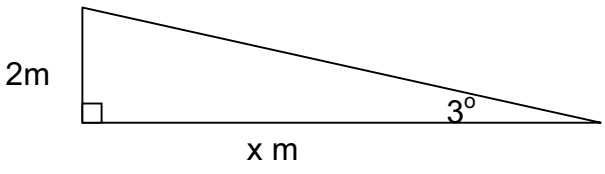
Important Note

To solve a problem in trigonometry, we will always need a picture. Even if the question gives you a diagram, you should always draw a simple right-angled triangle that shows the given and requested information. Then, follow the steps outlined previously to solve for the unknown side or angle.

Topic 1: Solve for a Unknown Side in a Word Problem

Example 1:

A wheelchair ramp is to be built outside a building. The building code recommends that the ramp have an angle of elevation of 3^o. The entry of the building is 2 m above the ground. How far from the base of the building will the ramp start?



The 2 m side is opposite to the 3° angle, and the *x* m side is adjacent to the 3° angle. Because we have opposite and adjacent sides, we have to use a tangent ratio for our equation.

$$\tan 3^o = \frac{2\,\text{m}}{x\,\text{m}}$$

$$x \tan 3^o = 2$$

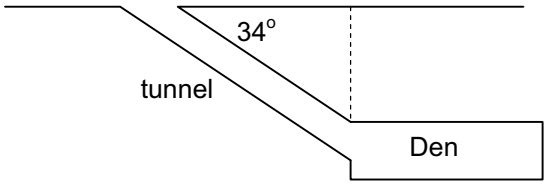
$$x = \frac{2}{\tan 3^o}$$

$$x = 38.2\,\text{m}$$

Therefore, the ramp must start about 38.2 m from the base of the building.

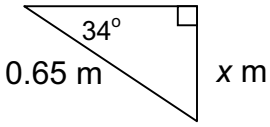
Example 2:

A mouse dug a tunnel at an angle of depression of 34°. The tunnel is 0.65 m long with a small den at the end. How far below the surface is the den?



We choose the variable, x, to represent how far the den is below the surface.

Here is a simpler picture, with labels:



$$\sin 34^{\circ} = \frac{x}{0.65}$$

$$0.65 \sin 34^{\circ} = x$$

$$x = 0.36$$

Therefore, the den is about 0.36 m below the surface.

The given side length is the hypotenuse; the requested side length is opposite the 34° angle. We must therefore use the sine ratio.

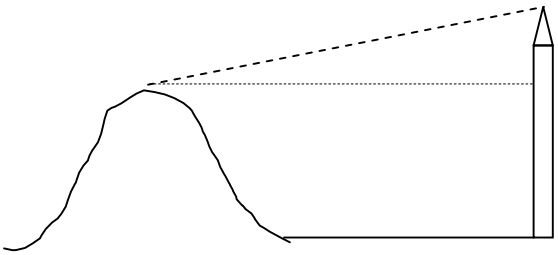
Assessment and Evaluation: Unit 4, Lesson 4

1. Draw a labelled diagram for each situation.

[a] A carpenter needs to cut a diagonal brace to support a rectangular frame that measures 85 cm wide by 160 cm long. What angle would be formed between the brace and the shorter side?

[b] A skateboarder wants to build a launch ramp, using a sheet of plywood 140 cm long for the angled surface of the ramp. She reads that the angle between the ground and the riding surface should be 25°. How high will the ramp be from ground to its top edge?

2. From the top of a hill 32.6 m high, the angle of elevation to the top of a TV relay tower is 13^0. The hilltop is 50.3 m from the tower. Determine the height of the tower.



Assessment and Evaluation: Unit 4, Lesson 4

3. A lighthouse is 75 m tall. The captain of a small boat drops anchor offshore from the base of the lighthouse. The lighthouse keeper measures an angle of depression of 22° from the top of the lighthouse to the boat.

   How far is the boat from the base of the lighthouse? Include a sketch, and draw the triangle you will use to solve the problem. State your solution.

4. An 8 m ladder leaning against a wall reaches a height of 7 m up the wall.

[a] What angle with the ground does the base of the ladder make?

[b] Use trigonometry to find the distance from the base of the wall to the base of the ladder.

[c] What other method could you have used to answer Question 4[b]?

5. In Example 1 from this lesson, we solved for the length of a wheelchair ramp.

[a] While the building code **recommends** that outdoor ramps should be at an angle of about 3°, the actual requirement **allows** a ramp with a slope of 1 part rise to 12 parts run.

Draw a right-angle triangle whose hypotenuse has this slope, and solve for the angle of elevation. Round your answer to the nearest tenth of a degree.

[b] How long would the ramp be if you were to use the angle you found in Part [a]?

[c] In our example, we found that the ramp had to be 38.2 m long. Unfortunately, the building code also says that the longest continuous length for one section of ramp is 9 m. Every 9 m, there must be a level landing, which is at least 1.2 m long.

Explain why you think the code includes this requirement, and explain how you would design an entrance ramp that meets the code requirements **and** respects the intention of a lower slope for an outdoor ramp.